Linear Algebra, Spring 2016
Quiz 11

Name: SOLUTIONS

You must show all your work to receive credit. Calculators are allowed. Open book and open notes, unlimited time. Do not, however, talk to anyone about it.

Problem 1: (3 points) Let U be the plane in $\mathbb{R}^3$ perpendicular to $\begin{bmatrix}1\\1\\1\end{bmatrix}$. Let $\mathrm{proj}_U : \mathbb{R}^3 \to \mathbb{R}^3$ be projection onto U. Find the matrix for proj_U.

Solution 1: $U = \text{null sp} \begin{bmatrix}1 & 1 & 1\end{bmatrix} = \text{span}\left\{ \underbrace{\begin{bmatrix}1\\-1\\0\end{bmatrix}}_{\vec{v}_1}, \underbrace{\begin{bmatrix}1\\0\\-1\end{bmatrix}}_{\vec{v}_2} \right\}$

use GS to turn into ONB:

$$\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \begin{bmatrix}1/\sqrt{2}\\-1/\sqrt{2}\\0\end{bmatrix}, \quad \vec{u}_2{}' = \vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1 = \begin{bmatrix}1\\0\\-1\end{bmatrix} - \frac{1}{\sqrt{2}}\vec{u}_1 = \begin{bmatrix}1/2\\1/2\\-1\end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{u}_2{}'}{\|\vec{u}_2{}'\|} = \begin{bmatrix}1/\sqrt{6}\\1/\sqrt{6}\\-2/\sqrt{6}\end{bmatrix}$$

matrix for proj_U is $\begin{bmatrix}\vec{u}_1 & \vec{u}_2\end{bmatrix}\begin{bmatrix}\vec{u}_1^{\,T}\\ \vec{u}_2^{\,T}\end{bmatrix} = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{6}\\-1/\sqrt{2} & 1/\sqrt{6}\\0 & -2/\sqrt{6}\end{bmatrix}\begin{bmatrix}1/\sqrt{2} & -1/\sqrt{2} & 0\\1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6}\end{bmatrix}$

$$= \begin{bmatrix}2/3 & -1/3 & -1/3\\-1/3 & 2/3 & -1/3\\-1/3 & -1/3 & 2/3\end{bmatrix}$$

Solution: Matrix is $A(A^TA)^{-1}A^T$, where A is ~~the~~ matrix with cols. a basis of U (does not need to be ONB!). (We didn't do this in class but some of you found this formula somewhere.)

Take $A = \begin{bmatrix}\vec{v}_1 & \vec{v}_2\end{bmatrix} = \begin{bmatrix}1 & 1\\-1 & 0\\0 & -1\end{bmatrix}$, then $A(A^TA)^{-1}A^T = \begin{bmatrix}2/3 & -1/3 & -1/3\\-1/3 & 2/3 & -1/3\\-1/3 & -1/3 & 2/3\end{bmatrix}$