Linear Algebra, Spring 2016
Quiz 11

Name: SOLUTIONS

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation $L(\vec{x}) = A\vec{x}$, with A the matrix

$$A = \begin{bmatrix} -2 & 3 & 4 \\ 4 & -5 & 1 \end{bmatrix}.$$

Find basis for Ker L and Range L.

$\text{Ker } L = \{ \vec{x} \mid A\vec{x} = \vec{0} \} = \text{null sp } A$

$\text{Range } L = \{ A\vec{x} \mid \vec{x} \in \mathbb{R}^3 \} = \text{col sp } A$

$$\text{rref}(A) = \begin{bmatrix} \textcircled{1} & 0 & 11.5 \\ 0 & \textcircled{1} & 9 \end{bmatrix}$$

x_3 free variable

$$\text{basis of Range } L = \left\{ \begin{bmatrix} -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \end{bmatrix} \right\}$$

$A\vec{x} = \vec{0} \Rightarrow$

$$x_1 = -11.5\,t$$
$$x_2 = -9t$$
$$x_3 = t$$

$$, \quad \vec{x} = t \begin{bmatrix} -11.5 \\ -9 \\ 1 \end{bmatrix}$$

$$\text{basis of Ker } L = \left\{ \begin{bmatrix} -11.5 \\ -9 \\ 1 \end{bmatrix} \right\}$$