Hint for #1

Use the equations to find a relationship between A_f and A_f':

$$A_f [\vec{v}]_B = [f(\vec{v})]_B \quad \text{(definition of } A_f\text{)}$$

$$A_f' [\vec{v}]_{B'} = [f(\vec{v})]_{B'} \quad \text{(definition of } A'_f\text{)}$$

$$T [\vec{v}]_B = [\vec{v}]_{B'} \quad \text{(T is change of coord. matrix, it is invertible; explicit formula for T is not needed)}$$

Also, if X, Y are matrices and $X\vec{v} = Y\vec{v}$ for all vectors $\vec{v}$, then $X = Y$. You might find this fact useful.

#5

The integral is over the region $0 \le r \le R$, $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi$. It should have been written as

$$\int_0^R \int_0^{2\pi} \int_0^{\pi} \left| \operatorname{Det} Df_{(r,\theta,\varphi)} \right| d\varphi \, d\theta \, dr$$