Math 3333-003
Spring 2016
Exam 1

Name: SOLUTIONS

Problem	Points
Problem 1 (3 pts)	
Problem 2 (3 pts)	
Problem 3 (3 pts)	
Problem 4 (3 pts)	
Problem 5 (3 pts)	
Problem 6 (3 pts)	
Problem 7 (3 pts)	
Problem 8 (3 pts)	
Total	

Instructions:

- Calculators are allowed. You can use your calculator to put matrices in rref, find inverses, and multiply matrices.
- You must show all of your non-calculator work to receive full credit. Exception: You do not need to show your work for matching and fill in the blank questions.
- Some problems are easier than others. Make sure to attempt all the problems before spending a lot of time on the hard ones.

1. (3 points) Solve the following system. Write your answer in column vector form, and use parameters if there is more than one solution.

$$\begin{cases} -2x_1 - 4x_2 + 3x_3 + 9x_4 = 1 \\ -x_1 - 2x_2 + x_3 + 5x_4 = 2 \\ 3x_1 + 6x_2 - 5x_3 - 13x_4 = 0 \\ -2x_1 - 4x_2 + 2x_3 + 10x_4 = 4 \end{cases}$$

rref of aug. matrix is $\begin{bmatrix} 1 & 2 & 0 & -6 & -5 \\ 0 & 6 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

x_2, x_4 free

s, t parameters

$x_1 = -5 - 2s + 6t$

$x_2 = s$

$x_3 = -3 + t$

$x_4 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ -3 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 6 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

2. (3 points) Find scalars x, y, z, w which satisfy the following vector equation. If there are no such scalars, write no solution.

$$x \begin{bmatrix} -3 \\ 2 \\ 1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 3 \\ 2 \\ 4 \\ -9 \end{bmatrix} + z \begin{bmatrix} 4 \\ -1 \\ 7 \\ 2 \end{bmatrix} + w \begin{bmatrix} 0 \\ 4 \\ 3 \\ -7 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 3 & 4 & 0 \\ 2 & 2 & -1 & 4 \\ 1 & 4 & 7 & 3 \\ 2 & -9 & 2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -3 & 3 & 4 & 0 \\ 2 & 2 & -1 & 4 \\ 1 & 4 & 7 & 3 \\ 2 & -9 & 2 & -7 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 1.7 \\ -.1 \\ -1.8 \end{bmatrix}$$

3. (3 points) For each of the following matrices, write "In rref" if it is in reduced row echelon form. If it is not, circle a non-zero entry which is causing it to not be in rref, and write down a row-operation you could perform to correct it.

a) $\begin{bmatrix} 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 4 & 6 \\ 0 & 1 & 0 & 0 & -2 \end{bmatrix}$ $R_2 \longleftrightarrow R_3$

b) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $R_2 \longrightarrow -R_2$

c) $\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $R_1 \longrightarrow R_1 - 2R_3$

4. (3 points) The augmented matrices of six different systems are given in a)-f). A geometric description of the systems is given in i)-vi), but in a different order. Fill in the blanks with the letter of the matrix that matches the geometric description.

a) $\left[\begin{array}{ccc|c} -1 & 2 & -3 & 4 \\ 2 & 7 & 0 & -1 \\ 1 & 9 & -3 & 2 \end{array}\right]$

b) $\left[\begin{array}{ccc|c} -3 & 4 & 1 & 10 \\ -\frac{3}{2} & 2 & \frac{1}{2} & 6 \end{array}\right]$

c) $\left[\begin{array}{cc|c} 4 & 2 & 4 \\ 2 & 1 & 3 \end{array}\right]$

d) $\left[\begin{array}{ccc|c} 2 & 5 & -3 & 4 \\ -1 & 3 & 6 & 1 \\ 0 & 2 & -3 & 2 \end{array}\right]$

e) $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \end{array}\right]$

f) $\left[\begin{array}{ccc|c} 0 & 1 & -2 & 10 \\ 5 & -7 & 1 & 0 \end{array}\right]$

i) Two parallel planes in $\mathbb{R}^3$. b

ii) Three lines in $\mathbb{R}^2$ intersecting in a single point. e

iii) Two parallel lines in $\mathbb{R}^2$. c

iv) Two planes in $\mathbb{R}^3$ intersecting in a line. f

v) Three planes in $\mathbb{R}^3$ intersecting in a single point. d

vi) Three planes in $\mathbb{R}^3$ with no common intersection points. a

5. (3 points) Give an example of matrices A and B such that $AB \neq BA$.

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}. \quad BA = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$$

6. (3 points) Suppose A and B are invertible matrices and

$$A^{-1} = \begin{bmatrix} 2 & 3 & 7 \\ -1 & 0 & 2 \\ 1 & 6 & 5 \end{bmatrix}, \qquad B^{-1} = \begin{bmatrix} 0 & -1 & -2 \\ 2 & 6 & 4 \\ 3 & 7 & 1 \end{bmatrix}.$$

Solve the equation

$$AB\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}.$$

$$AB\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad \cancel{A^{-1}A}B\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \Rightarrow B\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad B^{-1}B\vec{x} = B^{-1}A^{-1}\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad \vec{x} = B^{-1}A^{-1}\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -85 \\ 292 \\ 230 \end{bmatrix}$$

7. (3 points)

a) Let $L_A : \mathbb{R}^3 \to \mathbb{R}^3$ be the rotation that fixes the y-axis and rotates the xz-plane by π. What is the matrix A for this rotation?

$$\vec{e_1} \longmapsto -\vec{e_1}$$
$$\vec{e_2} \longmapsto \vec{e_2} \qquad \Rightarrow \quad A = \begin{bmatrix} -\vec{e_1} & \vec{e_2} & -\vec{e_3} \end{bmatrix}$$
$$\vec{e_3} \longmapsto -\vec{e_3}$$
$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

b) Let $L_B : \mathbb{R}^3 \to \mathbb{R}^3$ be the rotation that fixes the z-axis and rotates the xy-plane by $\pi/2$. What is the matrix B for this rotation?

$$\vec{e_1} \longmapsto \vec{e_2}$$
$$\vec{e_2} \longmapsto -\vec{e_1} \qquad \Rightarrow \quad B = \begin{bmatrix} \vec{e_2} & -\vec{e_1} & \vec{e_3} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\vec{e_3} \longmapsto \vec{e_3}$$

c) What is the matrix for the following tansformation: first perform the transformation of part a), then the transformation of part b).

$$L_B \circ L_A = L_{BA}$$

$$BA = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

8. (3 points) Let A be a 2x2 matrix, and let $\vec{v_1}$ and $\vec{v_2}$ denote the columns of A (so $A = \begin{bmatrix} \vec{v_1} & \vec{v_2} \end{bmatrix}$).

a) Fill in each blank with a number.

$$A \begin{bmatrix} \underline{2} & \underline{6} \\ \underline{-1} & \underline{3} \end{bmatrix} = \begin{bmatrix} 2\vec{v_1} - \vec{v_2} & 6\vec{v_1} + 3\vec{v_2} \end{bmatrix}$$

b) Now suppose

$$2\vec{v_1} - \vec{v_2} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}, \quad 6\vec{v_1} + 3\vec{v_2} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}.$$

Find A.

$$A \begin{bmatrix} 2 & 6 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ 4 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 6 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 2 \\ 1.5 & -1 \end{bmatrix}$$