Name: ______________________________

Differential Equations, Spring 2017 Quiz 10, April 28

You must show all your work to receive credit. Calculators are allowed.

**Problem 1:** (3 points) $A$ is a $3 \times 3$ matrix and $\vec{v_1}, \vec{v_2}, \vec{v_3}$ are vectors such that

$$A\vec{v_1} = -\vec{v_1}, \quad A\vec{v_2} = 2\vec{v_2}, \quad A\vec{v_3} = -4\vec{v_3}.$$

Find the solution of the IVP

$$\begin{aligned} \vec{x}' &= A\vec{x}, \\ \vec{x}(0) &= 7\vec{v_1} + 8\vec{v_2} + 9\vec{v_3}. \end{aligned}$$

Solution: the above information means that $\vec{v_1}, \vec{v_2}, \vec{v_3}$ are eigenvectors with eigenvalues $-1, 2, -4$. Since the eigenvalues are distinct, the eigenvectors are automatically linearly independent, so the general solution is

$$\vec{x}(t) = c_1 e^t \vec{v_1} + c_2 e^{2t} \vec{v_2} + c_3 e^{-4t} \vec{v_3}.$$

(Note that the system is 3 dimensional, so we know that the general solution is composed of 3 linearly independent solutions. If the system were higher dimensional we would not have enough information to write down the general solution.) Plugging in $t = 0$ gives $\vec{x}(0) = c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3}$. The IC then implies that $c_1 = 7, c_2 = 8, c_3 = 9$. Thus the solution to the IVP is

$$\vec{x}(t) = 7e^t \vec{v_1} + 8e^{2t} \vec{v_2} + 9e^{-4t} \vec{v_3}.$$