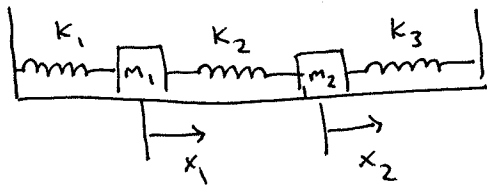


Section 4.1

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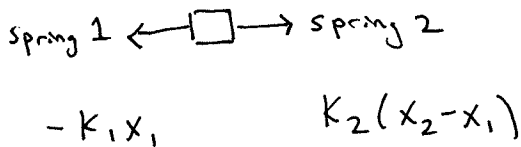


Spring 1 displ. = x_1

Spring 2 displ. = $x_2 - x_1$

Spring 3 displ. = $-x_2$

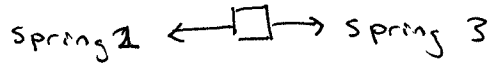
Forces on mass 1



$-k_1 x_1$

$k_2(x_2 - x_1)$

Forces on mass 2



$-k_2(x_2 - x_1)$

$-k_3 x_2$

Force = mass x acc.

$$m_1 x_1'' = -k_1 x_1 + k_2(x_2 - x_1) = -(k_1 + k_2)x_1 + k_2 x_2$$

$$m_2 x_2'' = -k_2(x_2 - x_1) - k_3 x_2 = k_2 x_1 - (k_2 + k_3)x_2$$

$$m_1 x_1'' = -(k_1 + k_2)x_1 + k_2 x_2$$

$$m_2 x_2'' = k_2 x_1 - (k_2 + k_3)x_2$$

Rewrite as 1st order system

$x_1 = x_3$

$x_2 = x_4$

$x_3 = x_1'$

$x_4 = x_2'$

\Rightarrow

$x_1' = x_3$

$x_2' = x_4$

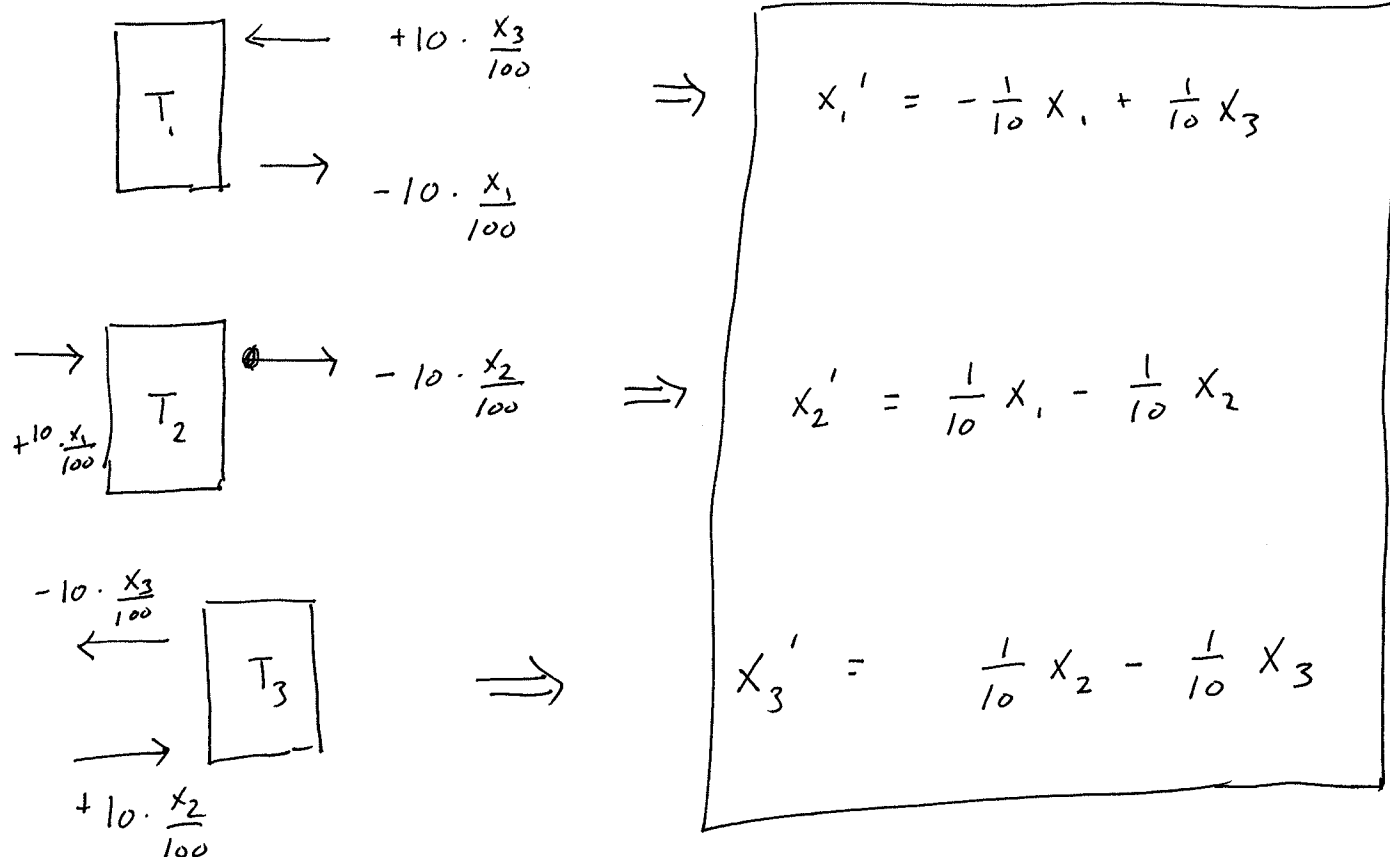
$x_3' = x_1'' = -\frac{k_1 + k_2}{m_1} x_1 + \frac{k_2}{m_1} x_2$

$x_4' = x_2'' = \frac{k_2}{m_2} x_1 - \frac{k_2 + k_3}{m_2} x_2$

let

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(2)



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position = $\vec{x} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

velocity = $\vec{v} = \begin{bmatrix} x' \\ y' \\ 0 \end{bmatrix}$

acceleration = $\vec{a} = \begin{bmatrix} x'' \\ y'' \\ 0 \end{bmatrix}$

$\vec{F} = q \vec{v} \times \vec{B} = q \begin{bmatrix} x' \\ y' \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix} = q \begin{vmatrix} i & j & k \\ x' & y' & 0 \\ 0 & 0 & B \end{vmatrix}$

cross product

$= q B (y' i - x' j) = \begin{bmatrix} q B y' \\ -q B x' \\ 0 \end{bmatrix}$

$\vec{F} = m \vec{a}$

 \Rightarrow

$\begin{bmatrix} q B y' \\ -q B x' \\ 0 \end{bmatrix} = \begin{bmatrix} m x'' \\ m y'' \\ 0 \end{bmatrix}$

 \Rightarrow

$\begin{bmatrix} m x'' = q B y' \\ m y'' = -q B x' \end{bmatrix}$

Rewrite as 1st order system

③

let

$$x_1 = x$$

$$x_2 = y$$

$$x_3 = x'$$

$$x_4 = y'$$

\Rightarrow

$$x_1' = x' = x_3$$

$$x_2' = y' = x_4$$

$$x_3' = x'' = \frac{q}{m} B y' = \frac{q}{m} B x_4$$

$$x_4' = y'' = -\frac{q}{m} B x' = -\frac{q}{m} B x_3$$

$$\begin{aligned} x_1' &= x_3 \\ x_2' &= x_4 \\ x_3' &= \frac{q}{m} B x_4 \\ x_4' &= -\frac{q}{m} B x_3 \end{aligned}$$