Differential Equations, Spring 2017
Written Assignment #3, Due Friday, March 3

Consider the equation

$$\frac{dP}{dt} = P(5 - P) - h,$$

with h a parameter (constant). We can think of this as a logisitic equation with an external pressure $-h$ acting on the population. To be specific, let $P(t)$ be thousands of fish in a pond, t be time in years. The $-h$ term can be thought of as arising from removal of h fish per year from the pond by fishing (or stocking of fish if $h < 0$). The equation is called the "logisitic equation with harvesting". The goal of this assignment is to understand what effect h has on the set of solutions of the ODE. In particular, the goal is to learn how the phase diagrams change when h changes, and to understand the corresponding qualitative changes in the solutions of the ODE.

Answer the following questions. You can write your answers on the back or on a separate sheet of paper.

1. (3 points) Let $f(P, h) = P(5 - P) - h$ be the right hand side of the ODE. We will think of f as a function of P and h. With P on the horizontal axis and h on the vertical axis, graph the solution to $f(P, h) = 0$ in the Ph-plane. Your graph should divide the plane into two regions. Put a $+$ sign in the region where $f > 0$ and a $-$ sign in the region where $f < 0$.

2. (3 points) Draw three horizontal lines on your graph, one which intersects the curve $f = 0$ in two points, one which intersects the curve $f = 0$ in one point, and one which does not intersect the curve $f = 0$. Draw dots at the intersection points of the lines and the curve $f = 0$. Note that h is constant on a horizontal line, so you can think of each line as being a number line of P values. Now make each of the three horizontal lines a phase diagram for the equation $dP/dt = f(P, h)$ (so you will have 3 different phase diagrams, each one corresponding to a different value of h). To do this, all you have to do is draw arrows indicating the direction of flow. The $+$ and $-$ signs you drew in in the previous problem should make it very easy to do this.

3. (3 points) You should now begin to be able to see how the phase diagrams for the ODE change as h changes. Try to visualize how they change by imagining one of the phase diagrams from the previous problem moving up a little bit or down a little bit (moving up or down a little bit corresponds to changing h a little bit). You should be able to see that there is an exceptional h value at which the nature of the phase diagram completely changes: the phase diagrams slightly below the exceptional value will be qualitatively different than the phase diagrams slightly above the exceptional value. Let h_0 stand for the exceptional value of h. What is h_0? In terms of what is happening to fish in the pond, h_0 has a special meaning. What is the fate of the fish population in the pond if the fishing rate h exceeds the exceptional value h_0?