You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) The ODE for a pendulum is

$$m\theta'' + \frac{mg}{L}\sin\theta = 0,$$

where $\theta = \theta(t)$ is the angle between the pendulum and the vertical direction, m is the mass (a constant), g is acceleration due to gravity (a constant), and L is the length of the pendulum (a constant). Let $x_1 = \theta$ and $x_2 = \theta'$. Prove that the function

$$H(x_1, x_2) = \frac{mL^2 x_2^2}{2} + mgL(1 - \cos x_1)$$

is a conserved quantity for the system. (It is in fact the energy but you don't need to show this.)

$$\frac{d}{dt}\left(H(x_1(t), x_2(t))\right) = mL^2 x_2 x_2' + mgL(\sin x_1)x_1'$$

$$= mL^2 \theta'\theta'' + mgL(\sin\theta)\theta'$$

$$= mL^2\theta'\left(m\theta'' + \frac{mg}{L}\sin\theta\right)$$

$$= L^2\theta' \cdot 0 = 0$$