

Quiz 7, April 7

Find the general solution of

$$y''' + y'' = e^x + x$$

char eqn: $r^3 + r^2 = 0$

$$r^2(r+1) = 0$$

roots: $0, -1$

$$\Rightarrow y_c = C_1 x + C_2 + C_3 e^{-x}$$

mult: $2, 1$

nonhomog. term = $e^x + x$

lin. ind terms: $e^x, x, 1$

controlling roots: $1, 0, 0$

$$\Rightarrow y_p = A e^x + x^2 (Bx + C)$$

correction factor: x^0, x^2, x^2

$$y_p = A e^x + B x^3 + C x^2$$

$$y_p' = A e^x + 3B x^2 + 2C x$$

$$y_p'' = A e^x + 6B x + 2C$$

$$y_p''' = A e^x + 6B$$

$$\Rightarrow A e^x + 6B + A e^x + \cancel{3B x^2} + 2C x = e^x + x$$

$$2A e^x + 6B x + 6B + 2C = e^x + x$$

$$\Rightarrow \begin{aligned} 2A &= 1 & \Rightarrow A &= 1/2 \\ 6B &= 1 & \Rightarrow B &= 1/6 \\ 6B + 2C &= 0 & C &= -1/2 \end{aligned}$$

\Rightarrow

$$y = \frac{1}{2} e^x + \frac{1}{6} x^3 - \frac{1}{2} x^2 + C_1 x + C_2 + C_3 e^{-x}$$