Name: _____

Differential Equations, Spring 2017 Quiz 6, March 10

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Here are the characteristic equations of some linear ODEs with constant coefficients. For each one, write down the general solution.

a) $(r-1)^2(r^2+5)=0$ $\quad r = 1, \pm i\sqrt{5}$ (1 double)

$$y = C_1 e^x + C_2 x e^x + C_3 \cos(\sqrt{5}x) + C_4 \sin(\sqrt{5}x)$$

b) $r^3+6r^2+4r=0$

$r(r^2+6r+4)=0$, $r=0$, $r = \frac{-6 \pm \sqrt{36-16}}{2} = -3 \pm \frac{\sqrt{20}}{2}$

$= -3 \pm \sqrt{5}$

$$y = C_1 e^{0x} + C_2 e^{(-3+\sqrt{5})x} + C_3 e^{(-3-\sqrt{5})x}$$

c) $(r^2-r-12)(r^2+r+1)^2=0$

$(r-4)(r+3)$, $r = 4, -3$; $r = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$, each one is double

$$y = C_1 e^{4x} + C_2 e^{-3x} + e^{-\frac{1}{2}x}\left[(C_3 + x C_4)\cos\left(\frac{\sqrt{3}}{2}x\right) + (C_5 + x C_6)\sin\left(\frac{\sqrt{3}}{2}x\right)\right]$$