Name: Solution

Differential Equations, Spring 2017 Quiz 4, Feb 24

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Find the implicit general solution to the equation

$$\frac{dy}{dx} = \frac{-1 - ye^{xy}}{2y + xe^{xy}}.$$

$(2y + xe^{xy})\,dy = (-1 - ye^{xy})\,dx \Rightarrow (1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0$

$\downarrow \partial/\partial y$ $\downarrow \partial/\partial x$

$e^{xy} + xye^{xy}$ $e^{xy} + xye^{xy}$

are equal, so is exact.

Find $H(x,y)$ with

$$\begin{cases} \frac{\partial H}{\partial x} = 1 + ye^{xy} \\ \frac{\partial H}{\partial y} = 2y + xe^{xy} \end{cases}$$

$$\Rightarrow H = \int \frac{\partial H}{\partial x}\,dx = \int (1 + ye^{xy})\,dx = x + e^{xy} + C(y)$$

$$\Rightarrow 2y + xe^{xy} = \frac{\partial H}{\partial y} = xe^{xy} + C'(y)$$

$$\Rightarrow C'(y) = 2y, \quad C(y) = y^2, \quad H = x + e^{xy} + y^2$$

$\Rightarrow$ implicit solution is

$$\boxed{x + e^{xy} + y^2 = C}$$