Homework #1

Section 1.1, #43

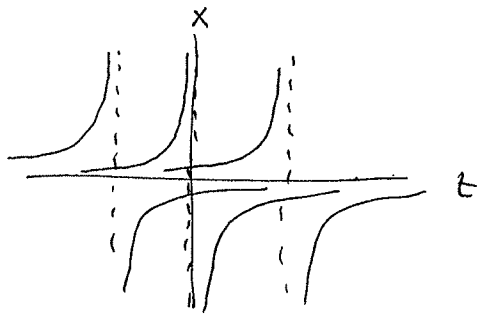
a) $x(t) = \dfrac{1}{c-kt}$

$$\frac{dx}{dt} = \frac{-1 \cdot -k}{(c-kt)^2} = k \cdot \left(\frac{1}{c-kt}\right)^2 = kx^2$$

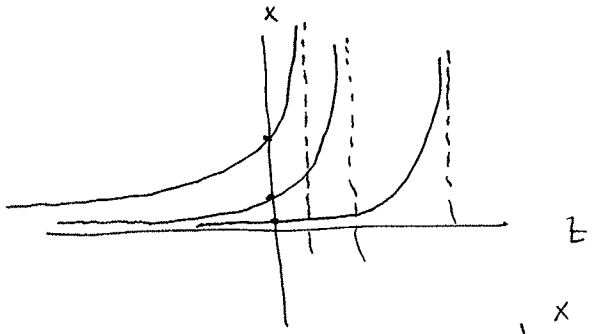
b) $x(t) = 0$ works : $\dfrac{dx}{dt} = 0$, $kx^2 = 0$, $x(0) = 0$

#44

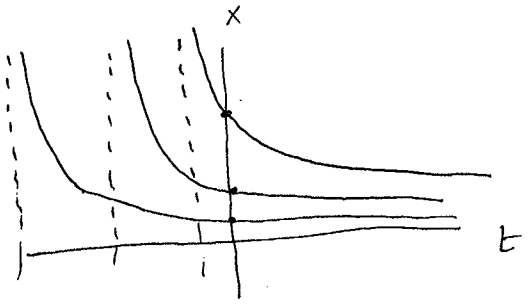
a) graphs of $x(t) = \dfrac{1}{c-kt}$: ($k > 0$)



for $x(0) > 0$, we only take the curves passing through the positive x-axis:



b) for $k < 0$, get



#45

$\frac{dP}{dt} = kP^2$, $P(0) = 2$, $P' = 1$ when $P = 10$

$\Rightarrow \quad 1 = k \cdot 10^2$

$k = 1/100$

so IVP is

$$\begin{cases} \frac{dP}{dt} = \frac{1}{100} P^2 \\ P(0) = 2 \end{cases}$$

general soltn. is $P(t) = \frac{1}{C - \frac{t}{100}}$

IC $\Rightarrow \quad 2 = P(0) = \frac{1}{C}$, $C = \frac{1}{2}$

so $\boxed{P(t) = \frac{1}{\frac{1}{2} - \frac{t}{100}}}$

Pop. is 100 when $P(t) = 100$: $\quad 100 = \frac{1}{\frac{1}{2} - \frac{t}{100}}$

$50 - t = 1 \quad \boxed{t = 49}$

Pop. is 1000 when $P(t) = 1000$: $\quad 1000 = \frac{1}{\frac{1}{2} - \frac{t}{100}}$

$500 - 10t = 1 \quad \boxed{t = 49.9}$

Pop. $\to \infty$ as $t \to 50$