Math 3113
Spring 2017
Exam 1, Feb 17

Name: SOLUTIONS

Problem	Points
Problem 1 (3 pts)	
Problem 2 (3 pts)	
Problem 3 (3 pts)	
Problem 4 (3 pts)	
Problem 5 (3 pts)	
Total	

Instructions:

- Calculators are allowed. No cell phones allowed.
- You must show all of your to receive credit.
- Some problems are easier than others. Make sure to attempt all the problems before spending a lot of time on the hard ones.

1. (3 points) Solve the initial value problem

$$\begin{aligned} \frac{dy}{dx} &= e^y(x^2+1) \\ y(2) &= 0. \end{aligned}$$

$$e^{-y}\,dy = (x^2+1)\,dx$$

$$-e^{-y} = \frac{1}{3}x^3 + x + C$$

$$e^{-y} = C - \frac{1}{3}x^3 - x$$

$$+y = -\ln\left(C - \frac{1}{3}x^3 - x\right)$$

IC: $\begin{matrix} x=2 \\ y=0 \end{matrix} \Rightarrow \quad 0 = -\ln\left(C - \frac{8}{3} - 2\right)$

$$C - \frac{8}{3} - 2 = 1$$

$$C = 1 + 2 + \frac{8}{3} = \frac{17}{3}$$

$$\boxed{y = -\ln\left(\frac{17}{3} - \frac{1}{3}x^3 - x\right)}$$

2. (3 points) Find the general solution of the equation

$$(1+x)y' = \ln x - y.$$

(You may assume $x > 0$.)

$$\frac{dy}{dx} + \frac{1}{1+x}\,y = \frac{\ln x}{1+x}$$

$$I(x) = e^{\int \frac{1}{1+x}\,dx} = e^{\ln|1+x|} = |1+x| = 1+x \quad \left(\begin{matrix}\text{since} \\ x>0\end{matrix}\right)$$

$$\Rightarrow \quad \frac{d}{dx}\left(y(1+x)\right) = \ln x$$

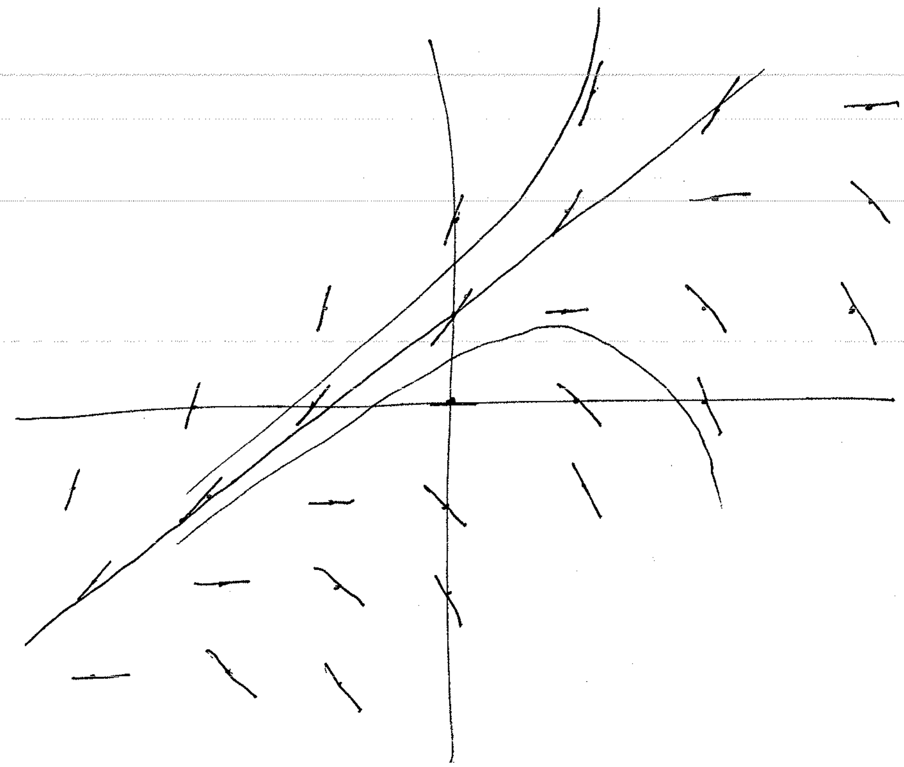
$$y(1+x) = \int \ln x \, dx = x\ln x - x + C$$

$$\boxed{y = \frac{x\ln x - x + C}{1+x}}$$

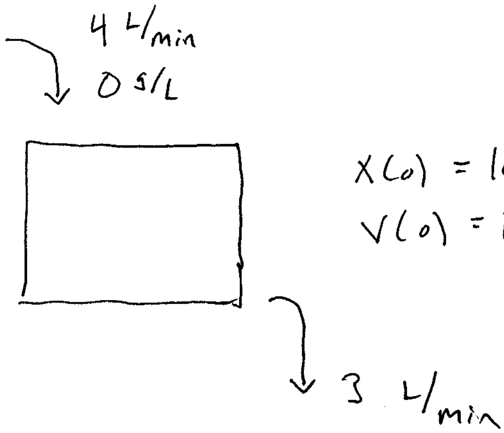
3. (3 points) Draw a slope field for the equation

$$\frac{dy}{dx} = y - x.$$

Use the slope field to sketch some representative solutions. (Do this without explicitly solving the equation.)



4. (3 points) A tank initially contains 10 L of water with 10 g of salt dissolved in it. Pure water starts to flow into the tank at a rate of 4 L/min. Thoroughly mixed water flows out of the tank at 3 L/min. Let $x(t)$ denote the mass of the salt inside the tank at time t. x is measured in g and t is measured in min. At what time are there 5 g of salt left in the tank?



$$X(0) = 10\text{ g}$$
$$V(0) = 10\text{ L}$$

$$V(t) = 10 + t$$

$$\frac{dx}{dt} = 0 - 3 \cdot \frac{x}{10+t}$$

$$\frac{dx}{dt} + \frac{3}{10+t} \cdot x = 0$$

$$I(t) = e^{\int \frac{3}{10+t}\,dt} = (10+t)^3$$

$$\Rightarrow \quad \frac{d}{dt}\left(x(10+t)^3 \right) = 0$$

$$x(10+t)^3 = C$$

$$x = \frac{C}{(10+t)^3}$$

$$IC: \quad 10 = x(0) = \frac{C}{10^3}$$

$$C = 10^4$$

$$x = \frac{10{,}000}{(10+t)^3}$$

$x(t) = 5$ when:

$$5 = \frac{10{,}000}{(10+t)^3}$$

$$(10+t)^3 = 2000$$

$$\boxed{t = 2000^{1/3} - 10}$$

5. (3 points) A population of bacteria satisfies the law of natural growth. It is observed that the population is growing at a rate of 100 bacteria per minute when there are 50 bacteria. How long does it take the population to double?

$$\frac{dP}{dt} = KP$$

when $P = 50$, $\frac{dP}{dt} = 100 \Rightarrow K = 2$

So $$\frac{dP}{dt} = 2P$$

$$P(t) = P_0 e^{2t}$$

doubles when $2P_0 = P_0 e^{2t}$

$$2 = e^{2t}$$

$$\boxed{t = \frac{\ln 2}{2}}$$