Differential Equations, Fall 2016 Quiz 8
Name: _____ Section:

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Find the general solution to

$$y'' + 9y = te^t.$$

Solution:
The characterstic equation is $r^2 + 9 = 0$, which has roots $\pm 3i$, so the complementary solution is

$$y_c = C_1 \cos(3t) + C_2 \sin(3t).$$

Look for a particular solution of the form

$$y_p = e^t(At + B).$$

Then

$$\begin{gathered} y_p = e^t(At + B) \\ y_p' = e^t(At + B) + e^t A = e^t(At + A + B) \\ y_p'' = e^t(At + A + B) + e^t A = e^t(At + 2A + B) \\ y_p'' + 9y_p = e^t(At + 2A + B + 9At + 9B) = 10Ate^t + (2A + 11B)e^t \end{gathered}$$

We need this to equal te^t, so

$$\begin{gathered} 10A = 1, \quad 2A + 11B = 0 \\ A = 1/10, \quad B = -1/55 \\ y_p = (t/10 - 1/55)e^t. \end{gathered}$$

Thus the general solution is

$$y = (t/10 - 1/55)e^t + C_1 \cos(3t) + C_2 \sin(3t).$$