Differential Equations, Fall 2016   Quiz 10

Name: _____ Section:

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Suppose A is a 2×2 matrix such that $\lambda = 2 + 3i$ is an eigenvalue with eigenvector

$$\vec{v} = \begin{bmatrix} i \\ 1 + i \end{bmatrix}.$$

Find the general solution to the linear system

$$\vec{x}' = A\vec{x}.$$

Solution: Two linearly independent real-valued solutions are the real and imaginary parts of the complex valued solution

$$
\begin{aligned}
e^{(2+3i)t}\vec{v} &= e^{2t}(\cos 3t + i \sin 3t)\begin{bmatrix} i \\ 1 + i \end{bmatrix} \\
&= e^{2t}\begin{bmatrix} -\sin 3t + i \cos 3t \\ \cos 3t - \sin 3t + i \sin 3t + i \cos 3t \end{bmatrix} \\
&= e^{2t}\begin{bmatrix} -\sin 3t \\ \cos 3t - \sin 3t \end{bmatrix} + ie^{2t}\begin{bmatrix} \cos 3t \\ \sin 3t + \cos 3t \end{bmatrix}.
\end{aligned}
$$

So the general solution is

$$\vec{x} = C_1 e^{2t}\begin{bmatrix} -\sin 3t \\ \cos 3t - \sin 3t \end{bmatrix} + C_2 e^{2t}\begin{bmatrix} \cos 3t \\ \sin 3t + \cos 3t \end{bmatrix}.$$