Differential Equations, Fall 2016 Quiz 6

Name: _____ Section:

You must show all your work to receive credit. Calculators are allowed. The quiz is due in class on Monday. It is closed book and closed notes. You are not allowed to talk to anybody about it. There is no time limit.

Problem 1: (3 points) Suppose a mass is attached to a horizontal spring and is oscillating back and forth. The mass is m kg, the spring constant is k N/m, and the amplitude of the oscillation is C m. Recall that the energy of the system is

$$E = \frac{mv^2}{2} + \frac{kx^2}{2},$$

where x is the displacement of the spring and $v = x'$. Prove that $E = kC^2/2$.

Solution 1: Since E = constant, can calculate E at any instant.

At the instant when displacement is maximized, $x = C$ and $v = x' = 0$.

Thus $E = \frac{m \cdot 0^2}{2} + \frac{k \cdot C^2}{2} = \frac{kC^2}{2}$.

Solution 2: General solution is $x = \pm C\cos\beta$ with $\beta = \sqrt{\frac{k}{m}}t - \alpha$.

Then:

$$E = \frac{m(x')^2}{2} + \frac{kx^2}{2} = \frac{m\left(-\sqrt{\frac{k}{m}} \pm C\sin\beta\right)^2}{2} + \frac{k(C\cos\beta)^2}{2}$$

$$= \frac{kC^2\sin^2\beta}{2} + \frac{kC^2\cos^2\beta}{2} = \frac{kC^2}{2}\left(\sin^2\beta + \cos^2\beta\right) = \frac{kC^2}{2}$$