Solution

Differential Equations, Spring 2016 — Quiz 3, Sep 16

Name: ______________________ Section: __________

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) A certain lake has a volume of V liters. A factory dumps polluted water into the lake at a rate of R_F liters per second. The concentration of the pollutant in the polluted water is C grams per liter. Clean water flows into the lake through intake streams at a rate of R_C liters per second. Thoroughly mixed water flows out of the lake through output streams. Assume that the volume of the lake is constant in time. Let $Q(t)$ denote the amount of pollutant in the lake as a function of time. Answer the following questions:

a) Write down the differential equation that $Q(t)$ satisfies.

b) What value does the concentration of the pollutant in the lake approach as time goes to infinity? Answer this question without explicitly solving the equation, and give some justification for your answer.

a) Volume constant $\Rightarrow$ rate out = rate in = $R_F + R_c$

$$\frac{dQ}{dt} = (\text{in}) - (\text{out}) = R_F \cdot C - (R_F + R_c) \cdot \frac{Q}{V} \quad g/\text{sec}$$

$$\boxed{\frac{dQ}{dt} = R_F \cdot C - (R_F + R_c) \cdot \frac{Q}{V}}$$

b) equilibrium solution occurs when $R_F C - (R_F + R_c) \cdot \frac{Q}{V} = 0$

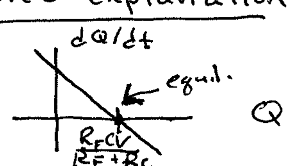
$\Rightarrow \quad Q = \dfrac{R_F C V}{R_F + R_c}$ is equilibrium amount

equilibrium concentration $= \dfrac{Q}{V} = \boxed{\dfrac{R_F C}{R_F + R_c}}$

Short explanation

all solutions approach equilibrium solution as $t \to \infty$

detailed explanation



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