Math 3113-007
Fall 2016 Exam 2

Name:____

Problem	**Points**
Problem 1 (3 pts)	
Problem 2 (3 pts)	
Problem 3 (3 pts)	
Problem 4 (3 pts)	
Problem 5 (3 pts)	
Total	

Instructions:

- Calculators are allowed. No cell phones allowed.
- You must show all of your to receive credit.
- Some problems are easier than others. Make sure to attempt all the problems before spending a lot of time on the hard ones.

1. (3 points) Find the general solution of the equation

$$y^{(4)} - 16y = 0$$

char. eqtn. $r^4 - 16 = 0$

$(r^2 - 4)(r^2 + 4) = 0$

$r = \pm 2, \pm 2i$

$$y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos 2x + C_4 \sin 2x$$

3. (3 points) Consider a horizontal spring with no damping. The spring exerts a force of 5 N when stretched 7 m. A mass of 10 kg is attached to the spring. At time $t = 0$ the spring is given an initial displacement of 2 m and an initial velocity of -1 m/sec. Find the displacement $x(t)$ as a function of time t.

$$5 = 7K \Rightarrow K = \frac{5}{7}$$

$$\begin{cases} 10x'' + \frac{5}{7}x = 0 \\ x(0) = 2 \\ x'(0) = -1 \end{cases}$$

$$x'' + \frac{1}{14}x = 0 \Rightarrow x = C_1 \cos\frac{t}{\sqrt{14}} + C_2 \sin\frac{t}{\sqrt{14}}$$

$$2 = x(0) = C_1$$

$$-1 = x'(0) = \frac{C_2}{\sqrt{14}}$$

$$\Rightarrow \boxed{x = 2\cos\frac{t}{\sqrt{14}} - \sqrt{14}\sin\frac{t}{\sqrt{14}}}$$

Or:

$$x = C\cos\left(\frac{t}{\sqrt{14}} - \alpha\right)$$

$$C = \sqrt{18}$$

$$\alpha = -\sin^{-1}\sqrt{\frac{14}{18}}$$

$$2 = x(0) = C\cos\alpha$$

$$-1 = x'(0) = +\frac{C}{\sqrt{14}}\sin\alpha$$

$$\boxed{x = \sqrt{18}\cos\left(\frac{t}{\sqrt{14}} + 1.079\right)}$$

2. (1 point each) For each question, set up an appropriate form for the particular solution y_p that you would look for when using the method of undetermined coefficients. You do not need to find the values of the coefficients.

   (a) $y'' + 2y' + y = xe^{-x}$

   $r^2 + 2r + 1 = 0$

   $(r+1)^2 = 0$

   $y_p = x^2 e^{-x}(Ax + B)$

   (b) $y'' + y = \cos x + \sin x$

   $r^2 + 1 = 0$

   $r = \pm i$

   $y_p = x(A\cos x + B\sin x)$

   (c) $y'' + 2y' = 1 + e^{-2x}\sin x$

   $r^2 + 2r = 0$

   $r(r+2) = 0$

   $y_p = x \cdot A + e^{-2x}(B\cos x + C\sin x)$

4. (3 points) Suppose $x_1(t)$ and $x_2(t)$ satisfy the same second-order, linear, homogeneous differential equation (on all of $\mathbb{R}$). Furthermore, suppose

$$x_1(0) = 1, \quad x_2(0) = 2, \quad x_1'(0) = 3, \quad x_2'(0) = 6.$$

If $x_1(5) = 7$, what is $x_2(5)$? Explain your reasoning. If there is not enough information to determine this, write "not enough information", and explain why.

$x_1(0) = 1$ $\quad$ $x_2(0) = 2$

$x_1'(0) = 3$ $\quad$ $x_2'(0) = 6$

Since the ODE is homog., linear, $2X_1$ is a soltn.

Also, $2X_1$ satisfies the same IC as X_2.

So by uniqueness of solutions, $2X_1 = X_2$.

Thus $X_2(5) = 2X_1(5) = 2 \cdot 7 = 14$

$$\boxed{X_2(5) = 14}$$

5. (3 points) Consider a spring with spring constant $k = 2$ N/m, damping constant $c = 1$ N sec/m, and mass $m = 5$ kg attached. An external force $F(t)$ is also applied. The resulting equation of motion is

$$5x'' + x' + 2x = F(t).$$

The system is given some initial conditions. It is observed that

$$\lim_{t\to\infty} (x(t) - 2\sin t) = 0.$$

What is one possible $F(t)$ that would produce this behavior?

$$X(t) = X_c(t) + X_p(t).$$

For damped springs, $\lim_{t\to\infty} X_c(t) = 0$.

Thus $0 = \lim_{t\to\infty} (X(t) - 2\sin t) = \lim_{t\to\infty} (X_p(t) - 2\sin t)$

So $X_p(t) = 2\sin t$ will work.

Then $F(t) = 5X_p'' + X_p' + 2X_p$

$= -10\sin t + 2\cos t + 4\sin t$

$$\boxed{F(t) = -6\sin t + 2\cos t}$$