Math 3113-004
Fall 2016 Exam 2

SOLUTIONS

Name:_____

Problem	Points
Problem 1 (3 pts)	
Problem 2 (3 pts)	
Problem 3 (3 pts)	
Problem 4 (3 pts)	
Problem 5 (3 pts)	
Total	

Instructions:

- Calculators are allowed. No cell phones allowed.
- You must show all of your to receive credit.
- Some problems are easier than others. Make sure to attempt all the problems before spending a lot of time on the hard ones.

1. (3 points) Find the general solution of the equation

$$y^{(4)} + 4y^{(2)} = 0$$

char. eqn. $r^4 + 4r^2 = 0$

$r^2(r^2 + 4) = 0$

$r = 0, 0, \pm 2i$

$\Rightarrow \quad y = C_1 x + C_2 + C_3 \cos 2x + C_4 \sin 2x$

2. (1 point each) For each question, set up an appropriate form for the particular solution y_p that you would look for when using the method of undetermined coefficients. You do not need to find the values of the coefficients.

(a) $y'' - 2y' + y = e^x + 1$

$r^2 - 2r + 1 = 0$

$(r-1)^2$

$y_p = x^2 \cdot Ae^x + B$

(b) $y'' + y = \cos 2x + \sin x$

$r^2 + 1 = 0$

$r = \pm i$

$y_p = A \cos 2x + B \sin 2x + x(C \sin x + D \cos x)$

(c) $y'' - 5y' + 6y = e^{2x} \cos x + x^2$

$r^2 - 5r + 6 = 0$

$(r-3)(r-2) = 0$

$r = 2, 3$

$y_p = e^{2x}(A \cos x + B \sin x) + Cx^2 + Dx + E$

3. (3 points) Consider a horizontal spring with no damping. The spring exerts a force of 3 N when stretched 7 m. A mass of 5 kg is attached to the spring. At time $t = 0$ the spring is given an initial displacement of 1 m and an initial velocity of -3 m/sec. Find the displacement $x(t)$ as a function of time t.

$3 = 7 \cdot k, \quad k = 3/7$

$$\begin{cases} 5x'' + \frac{3}{7}x = 0 \\ x(0) = 1 \\ x'(0) = -3 \end{cases}$$

$x'' + \frac{3}{35}x = 0, \quad r^2 + \frac{3}{35} = 0, \quad r = \pm i\sqrt{\frac{3}{35}}$

$x = C_1 \cos\left(\sqrt{\frac{3}{35}}t\right) + C_2 \sin\left(\sqrt{\frac{3}{35}}t\right)$

$1 = x(0) = C_1$

$-3 = x'(0) = \sqrt{\frac{3}{35}}C_2$

$\Rightarrow \quad x = \cos\left(\sqrt{\frac{3}{35}}t\right) - 3\sqrt{\frac{35}{3}}\sin\left(\sqrt{\frac{3}{35}}t\right)$

or $\quad x = C\cos\left(\sqrt{\frac{3}{35}}t - \alpha\right)$

$1 = x(0) = C\cos\alpha$

$-3 = x'(0) = -C\sqrt{\frac{3}{35}}\sin(-\alpha)$

$C = \sqrt{106}$

$\alpha = -\tan^{-1}\sqrt{105}$

$x = \sqrt{106}\cos\left(\sqrt{\frac{3}{35}} + 1.47\right)$

4. (3 points) Suppose $x_1(t)$ and $x_2(t)$ satisfy the same second-order, linear, homogeneous differential equation (on all of $\mathbb{R}$). Furthermore, suppose

$$x_1(0) = 1, \quad x_2(0) = 2, \quad x_1'(0) = 3, \quad x_2'(0) = 3.$$

If $x_1(5) = 7$, what is $x_2(5)$? Explain your reasoning. If there is not enough information to determine this, write "not enough information", and explain why.

$x_1(0) = 1$ $\quad$ $x_2(0) = 2$

$x_1'(0) = 3$ $\quad$ $x_2'(0) = 3$

$$\text{Wronskian} = \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 3 - 6 = -3$$

So x_1 and x_2 are linearly independent.

Thus not enough information to determine $x_2(5)$.

5. (3 points) Consider a spring with spring constant $k = 2$ N/m, damping constant $c = 1$ N sec/m, and mass $m = 5$ kg attached. An external force $F(t)$ is also applied. The resulting equation of motion is

$$5x'' + x' + 2x = F(t).$$

The system is given some initial conditions. It is observed that

$$\lim_{t\to\infty}(x(t) - 2\cos t) = 0.$$

What is one possible $F(t)$ that would produce this behavior?

$x = x_p + x_c$

$\lim_{t\to\infty} x_c = 0$ for spring with damping

thus $0 = \lim_{t\to\infty}(x - 2\cos t) = \lim_{t\to\infty}(x_p - 2\cos t)$

$x_p = 2\cos t$ satisfies this. Then

$F(t) = 5x_p'' + x_p' + 2x = -10\cos t - 2\sin t + 4\cos t$

$= -6\cos t - 2\sin t$

$\boxed{F(t) = -6\cos t - 2\sin t}$