Chap. 4 + 5 Practice Problems

1) The 2nd order eqtn. for a spring is

$$m x'' + cx' + Kx = 0$$

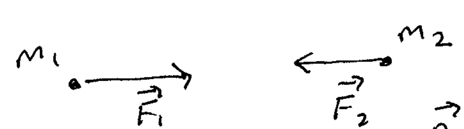
Write this as a 2 dim first order system.

2) The 2nd order eqtn. of a pendulum is

$$\theta'' + \frac{g}{L}\sin\theta = 0$$

Rewrite as 2d system.

3) Write the 2 body gravitational problem as a system (from Computer Project 4)



$\vec{F}_1, \vec{F}_2$ are gravitational forces

$$\vec{p}_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \vec{p}_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

$$\vec{p}_1\,' = \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} \qquad \vec{p}_2\,' = \begin{bmatrix} x_7 \\ x_8 \end{bmatrix}$$

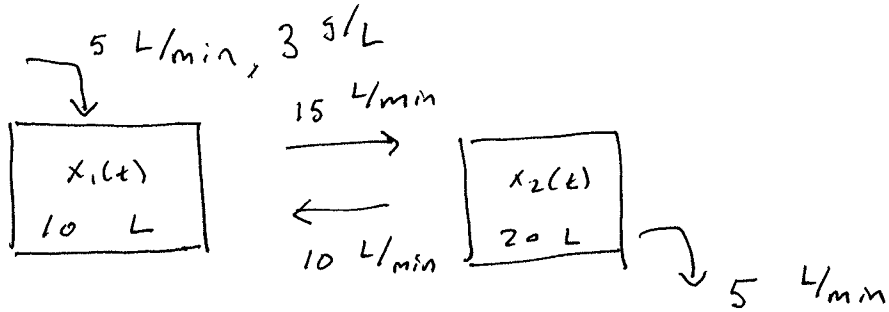
$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_8 \end{bmatrix}, \qquad \vec{x}\,' = ?$$

4) Is # 3 linear? Why or why not?

5) Let $A = n \times n$ matrix. If $\vec{x}_1 = \vec{x}_1(t)$ and $\vec{x}_2 = \vec{x}_2(t)$ are soltns. of $\vec{x}\,' = A\vec{x}$, show that $c_1\vec{x}_1 + c_2\vec{x}_2$ is also a soltn. for any constants c_1, c_2.

6) Draw the phase plane and vector field for the equation
$$\begin{cases} x_1' = -x_2 \\ x_2' = x_1 \end{cases}$$

7)



$x_1(0) = x_2(0) = 0$

Find $x_1(t)$, $x_2(t)$.

Find $\lim_{t\to\infty} x_1(t)$, $\lim_{t\to\infty} x_2(t)$.

Can you find these limits without solving any equations?
(Hint: Try to find an equilibrium solution of the system.)

8) Solve IVP
$$\begin{cases} x' = x - 2y \\ y' = 2x - 3y \\ x(0) = 1 \\ y(0) = 2 \end{cases}$$

9) Solve IVP
$$\begin{cases} x' = 2x + y \\ y' = x + 2y \\ x(0) = -1 \\ y(0) = 3 \end{cases}$$

10) Use eigenvalue method to solve

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

11)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 & -9 \\ 9 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

12)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

13) Are the vectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ lin ind?

what about $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$?

What about the functions $\vec{x}_1(t) = \begin{bmatrix} e^{t} \\ 2e^{t} \end{bmatrix}, \quad \vec{x}_2(t) = \begin{bmatrix} 2e^{2t} \\ 2e^{4t} \end{bmatrix}$?