

Chap. 3 Review

1) Find general soltn. of

$$x'' - 2x' + x = 0$$

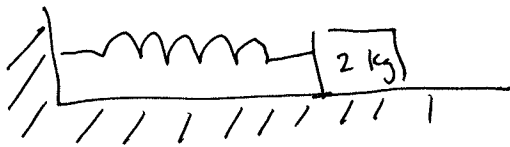
2)  $x'' - 2x' + 2x = 0$

3)  $x^{(4)} - x^{(3)} - 6x^{(2)} = 0$

4)  $x'' + 7x' + 12x = e^t$

5)  $x'' + x = \sin t + t^2$

6)



2 Kg mass on spring, no damping

When stretched 2 m, spring exerts force of 8 N.

Mass is given an initial displacement of 0 m and velocity of 10 m/s.

Find  $x(t)$  (displacement).

What is the period? Amplitude? Frequency? Energy?

- 7) Same setup as #6, except now there is a damping constant  $c$ . What values of  $c$  result in overdamping? Underdamping? Critical damping?
  
- 8) Suppose an external force  $F(t) = \sin(At)$  is applied to the spring in problem 6. What value of  $A$  will result in the energy of the system going to  $\infty$  as  $t \rightarrow \infty$ ?
  
- 9) Using force = mass  $\times$  acceleration, derive the ODE for a hanging spring.
  
- 10) For a system with damping, the damping constant  $c$  is positive. If  $c < 0$ , what happens to the amplitude as  $t \rightarrow \infty$ ?

11) Are the functions  $x_1(t) = e^t$ ,  $x_2(t) = e^{2t}$ ,  
 $x_3(t) = e^{4t}$  linearly independent? Why or why not?

12) Are the functions  $y_1(t) = \cos(t + \pi/4)$ ,  $y_2(t) = \sin t$ ,  
 $y_3(t) = \cos t$  linearly independent? Why or why not?