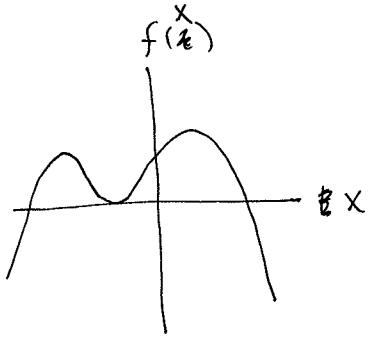
Chapters 1 + 2 Review

1) Draw a slope field for $\frac{dy}{dx} = x - y + 1$

Draw in some solution curves.

What happens as $x \to \infty$?

2)



Draw a phase diagram for the equation

$$\frac{dx}{dt} = f(x).$$

What happens to the critical points if the equation is changed to $\frac{dx}{dt} = f(x) + \varepsilon$? Are the critical points stable or unstable?

(ε is a small number).

3) Solve IVP $\begin{cases} \frac{dy}{dx} = ye^x \\ y(0) = 2e \end{cases}$

4) Solve IVP $\begin{cases} \frac{dy}{dx} = 2xy^2 + 3x^2y^2 \\ y(1) = -1 \end{cases}$

5) A population of bacteria follows the law of natural growth, $\frac{dP}{dt} = kP$. If the population doubles in 10 minutes, what is k? What are the units of k (assume t is measured in min)?

6) Give a plausible example of something that could be modelled with the equation $\frac{dx}{dt} = k(x - A)$, with A and k constants and $k > 0$.

7) Find the general solth. of $(y = y(x))$

$$y' = 1 + x + y + xy$$

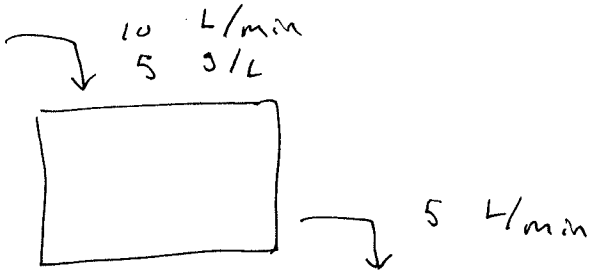
8) Find the general solth. of $(y = y(x))$

$$y' + y \cot x = \cos x$$

9) A tank initially contains 10 g of salt, 100 L of water.

Let $x(t)$ = amount of salt at time t.

Find $x(t)$.



10) Solve $x \frac{dy}{dx} + 6y = 3xy^{4/3}$ $(y = y(x))$

Hint: Bernoulli:

11) Solve $y' = (4x + y)^2$

Hint: Make a substitution.

12) Solve

$$(1 + ye^{xy})\,dx + (2y + xe^{xy})\,dy = 0$$

13) Solve $\dfrac{dy}{dx} = -\dfrac{2x+3y}{3x+2y}$

Find solth. ~~that~~ that passes through point $(x,y) = (1,2)$.

14) Recall that the Logistic Eqtn. is

$$\frac{dP}{dt} = k(M-P) \qquad (k, M \text{ positive constants}).$$

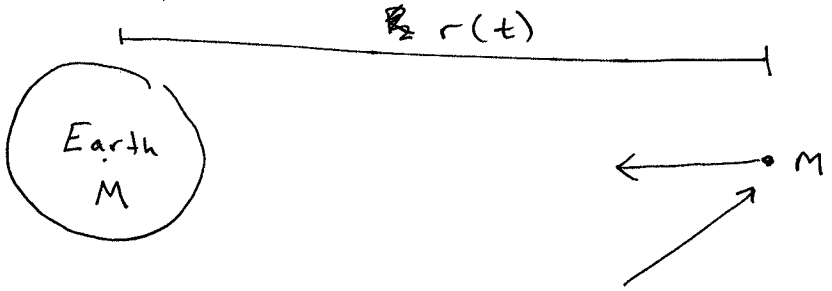
Draw the phase diagrams. If $P(0) > 0$, what is $\lim_{t\to\infty} P(t)$? Answer this without explicitly finding $P(t)$. What are the equilibrium solths.?

15) ~~What~~ If the population is harvested, so the eqtn. is

$$\frac{dP}{dt} = k(M-P) - H \qquad (H > 0)$$

what happens as H increases? What are the values of H that have special significance?

16)



force of gravity on the object is $F = \frac{GMm}{r^2}$

$\left(G \approx 6.6726 \times 10^{-11} \; N\left(m/kg\right)^2 \right)$

force = mass × acceleration $\Rightarrow$

$$\frac{GM}{r^2} = r''$$

Prove that $E = \frac{m(r')^2}{2} + \frac{GMm}{r}$

is constant along solutions

17) Suppose $r(0) = R$, $r'(0) = V_0$. (previous problem continued)

If $V_0 > \sqrt{\frac{2GM}{R}}$, show that

$$\lim_{t \to \infty} r(t) = \infty$$

($\sqrt{\frac{2GM}{R}}$ = escape velocity)