

Review Problems

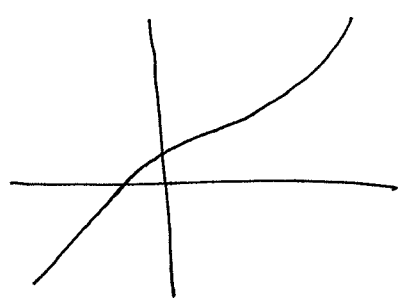
①

Ch. 6

1) If f has inverse f^{-1} , and $f(1) = 2$, $f'(1) = 10$,
find $(f^{-1})'(2)$.

2) If $f(x) = \frac{1-\sqrt{x}}{1+\sqrt{x}}$, find a formula for $f^{-1}(x)$.

3) Here is the graph of $f(x)$:



Does f have an inverse? If so, graph $f^{-1}(x)$.

4) If $f(x) = \int_3^x \sqrt{1+t^2} dt$, does f have an inverse?

Why or why not? If it does have an inverse, find $(f^{-1})'(0)$.

5) Suppose $f(g(x)) = x$ for all $x \in \mathbb{R}$.

(2)

Does f have an inverse? If so, prove it. If not, find a counterexample.

6) Use the definition of $\ln x$ ($\ln x = \int_1^x \frac{1}{t} dt$)

to prove that $\ln(xy) = \ln x + \ln y$.

7) Find the derivatives of the following functions:

a) $\ln(\sin x + x^2)$

b) $x^2 \log_5(2x-3)$

c) $\frac{\sqrt{x-10} \cdot \sin(5x^3)}{(10x+2)^3}$

d) x^x

e) $\ln\left(\frac{a-x}{a+x}\right)$

8) If $y = \ln 35$, find e^y .

(3)

9) Suppose $y = y_0 e^{kx}$ satisfies

$$y' = 10y, \quad y_0 = 100.$$

Find y_0 and k .

10) Find the derivatives of:

a) $e^{2x^2 + 5}$

b) 2^{x+10}

c) $e^{2x} e^{3x} e^{5x} 2^{10x}$

d) $\sqrt{x+2}^{\sin x}$

11) Find the integrals:

a) $\int \tan x \, dx$

b) $\int \frac{e^x}{1+e^x} \, dx$

$$c) \int \frac{1+e^x}{e^x} dx$$

$$d) \int 3^x dx$$

$$e) \int \frac{\sin(\ln x)}{x} dx$$

$$f) \int \frac{1}{ax+b} dx$$

12) Use the fact that $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$

to show that $e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$.

13) Find $\lim_{n \rightarrow \infty} \frac{d}{dx} \left[\left(1 + \frac{x}{n}\right)^n \right]$.

14) Find $\int \left(\frac{1}{10}\right)^x dx$.

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15) A population of bacteria has 10 organisms at time $t=0$ and 1000 organisms at time $t=25$.
When will there be 100,000 organisms?

16) Find the unique solution to the equation
 $y' = -2y$ that satisfies $y(1) = 3$.

17) Find $\lim_{x \rightarrow \infty} \tan^{-1}(-x^3 + 2x)$.

18) Find derivatives of the following functions:

a) $\sin^{-1}(2x + e^x)$

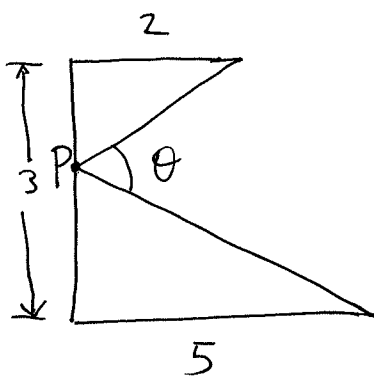
b) $(\cos^{-1} x)^2 + \tan^{-1} \frac{x}{a}$

c) $x \sec^{-1} x$

d) $\arccos \left(\frac{b + a \cos x}{a + b \cos x} \right)$

e) $\arcsin \left(\frac{x}{1+x} \right)$

19)



⑥

Where should the point P be chosen so as to maximize the angle θ ?

20) Suppose $0 \leq \theta \leq \pi$ and $\cos \theta = -\frac{4}{5}$.

Find $\sin \theta$ and $\tan \theta$.

21) Suppose $\theta = \cos^{-1} x$, where $-1 \leq x \leq 0$.

Find $\sin x$ and $\tan x$.

22) Find the integrals:

a) $\int \frac{dx}{1+a^2x^2}$

d) $\int \frac{1+x}{1+x^2} dx$

b) $\int \frac{dx}{a^2+x^2}$

e) $\int \frac{x}{1+x^4} dx$

c) $\int \frac{dx}{(\sin^{-1} x) \sqrt{1-x^2}}$

f) $\int \frac{dx}{x\sqrt{5-x^2}}$

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23) Use the definitions of $\sinh x$ and $\cosh x$ to prove the identity

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y.$$

24) Prove that $\cosh x$ has an inverse.

25) Find a formula for $\cosh^{-1} x$.

26) Use your answer from 25) to find $\frac{d}{dx}(\cosh^{-1} x)$.

27) Show that $y = A \sinh(Kx) + B \cosh(Kx)$ satisfies the equation $y'' = K^2 y$. Can you guess what the solution to $y'' = -K^2 y$ is?

28) Find $\lim_{x \rightarrow \infty} \tanh x$.

29) Find the following derivatives:

a) $\cosh^2 x - \sinh^2 x$

$$b) (\cosh x - \tanh x)^{10}$$

$$c) \operatorname{sech}^{-1} \frac{2x+5}{3x-10}$$

$$d) \tanh^{-1}(\cos x)$$

$$e) \tanh(\ln x + 5)$$

30) Find the following integrals:

$$a) \int \sinh x \cosh^2 x \, dx$$

$$b) \int \frac{\tanh(\ln x) \operatorname{sech}(\ln x)}{x} \, dx$$

$$c) \int (\cosh x + 1) e^{\sinh x + x} \, dx$$

$$d) \int_0^1 \frac{1}{\sqrt{16t^2 + 1}} \, dt$$

31) Find the following limits

$$a) \lim_{x \rightarrow \infty} \frac{e^x}{x^2}$$

$$h) \lim_{x \rightarrow 0^+} x \ln x$$

$$b) \lim_{x \rightarrow \infty} e^{-x} x^{1000}$$

$$i) \lim_{x \rightarrow 0^+} x^{\sin x}$$

$$c) \lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(5x)}$$

$$j) \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x$$

$$d) \lim_{x \rightarrow 0} \frac{1}{x} \left(\sqrt{1+ax} - 1 \right)$$

$$k) \lim_{x \rightarrow \infty} (x - \ln x)$$

$$e) \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{x - \frac{\pi}{2}}{\cos \frac{\pi}{2} x}$$

$$l) \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$$

$$d) \lim_{x \rightarrow \pi} \tan x$$

$$e) \lim_{t \rightarrow 1} \frac{t^8 - 1}{t^5 - 1}$$

$$f) \lim_{x \rightarrow 0} \frac{\sin^{-1}(kx)}{x}$$

$$g) \lim_{x \rightarrow 0} \frac{\cos x - 1}{2x^2}$$

32) Use integration by parts to prove

$$\int (\ln x)^n dx = x (\ln x)^n - n \int (\ln x)^{n-1} dx$$

33) Find the following integrals:

a) $\int \arctan x dx$

b) $\int x^2 e^x dx$

c) $\int x^3 e^{x^2} dx$

d) $\int t \sin(kt) dt$

e) $\int \ln(2x+5) dx$

f) $\int_1^3 y^3 \ln y dy$

g) $\int x^4 (\ln x)^2 dx$

h) $\int \cos \sqrt{x} dx$
(Hint: First make a u sub.)

34) Find the following integrals:

a) $\int \sin x (\cos x)^{-10} dx$

b) $\int \sin^2 x \cos^4 x dx$

c) $\int \sin(ax) \cos(bx) dx$

d) $\int \tan^4(2x) dx$

e) $\int \tan^3 x \sec x dx$

f) $\int \sec^3 x dx$

g) $\int \sec^3 x dx$

h) $\int \tan^3 x \sec^4 x dx$

35) Find the following integrals:

$$a) \int_0^a \sqrt{r^2 - x^2} \, dx \quad (0 \leq a \leq r)$$

$$b) \int x \sqrt{x^2 - a^2} \, dx$$

$$c) \int \frac{x}{(x^2 + a^2)^{3/2}} \, dx$$

$$d) \int \frac{dx}{(x^2 + 1)^2}$$

$$e) \int \frac{dx}{\sqrt{7 - x^2}}$$

$$f) \int \sqrt{x^2 + 2x} \, dx$$

$$g) \int x \sqrt{1 - x^4} \, dx$$

36) Find the integrals:

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$$a) \int \frac{x^4}{x-1} dx$$

$$b) \int \frac{1}{(x-1)(x+1)} dx$$

$$c) \int \frac{ax}{x^2-bx} dx$$

$$d) \int \frac{x^2+1}{(x-3)(x-2)^2} dx$$

$$e) \int \frac{1}{(x^2+9)^2} dx$$

$$f) \int \frac{x+4}{x^2+2x+5} dx$$

$$g) \int \frac{dx}{x(x^2+4)^2}$$