Math 2924-050
Fall 2014
Exam 2

Name:_____

Problem	**Points**
Problem 1 (20 pts)	
Problem 2 (15 pts)	
Problem 3 (15 pts)	
Problem 4 (15 pts)	
Problem 5 (15 pts)	
Problem 6 (20 pts)	
Total	

1. (20 points) Use a trig substitution to find

$$\int \sqrt{a^2 - x^2}\, dx$$

$x = a \sin\theta$, $\sqrt{a^2 - x^2} = a\cos\theta$

$dx = a\cos\theta$

$$\int \sqrt{a^2 - x^2}\, dx = \int a^2 \cos^2\theta\, d\theta = \frac{a^2}{2} \int (1 + \cos 2\theta)\, d\theta$$

$$= \frac{a^2}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) = \frac{a^2}{2}\left(\theta + \sin\theta\cos\theta\right)$$

$$= \frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{a}\sqrt{1 - \frac{x^2}{a^2}}\right)$$

$$= \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + C$$

2. (15 points) Find the limit

$$\lim_{u\to\infty}\left[u\tan^{-1}(u)-\frac{\pi}{2}u\right].$$

$$= \lim_{u\to\infty} \frac{\tan^{-1} u - \pi/2}{\frac{1}{u}} = \lim_{u\to\infty} \frac{\frac{1}{1+u^2}}{-\frac{1}{u^2}}$$

$$= \lim_{u\to\infty} \frac{-u^2}{1+u^2} = -1$$

3. (15 points) Find the unique function y that satisfies

$$y'' = 4y, \quad y(0) = 1, \quad y'(0) = 2.$$

Hint: Look for a solution of the form $y = A\sinh(mx) + B\cosh(mx)$ with A, B, m constants.

$$y'' = Am^2 \sinh(mx) + Bm^2 \cosh(mx)$$

$$4y = 4A \sinh(mx) + 4B \cosh(mx)$$

$y'' = 4y \Rightarrow m^2 = 4$, so take $m = 2$,

$$y = A \sinh 2x + B \cosh 2x$$

$$1 = y(0) = B$$

$$2 = y'(0) = \left(2A \cosh 2x + 2B \sinh 2x \right)\Big|_{x=0} = 2A$$

So $y = \sinh 2x + \cosh 2x$

4. (15 points) Find

$$\int \sin^{-1} x \, dx.$$

$u = \sin^{-1} x$, $dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} dx$, $v = x$

$= uv - \int v \, du$

$= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$

$= x \sin^{-1} x + \frac{1}{2} \int \frac{-2x}{(1-x^2)^{1/2}} dx$

$= x \sin^{-1} x + \frac{1}{2} \cdot \frac{1}{1/2} (1-x^2)^{1/2} + C$

$= x \sin^{-1} x + \sqrt{1-x^2} + C$

5. (15 points) Find

$$\int \frac{1}{x^2-1}\,dx.$$

$$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$1 = A(x+1) + B(x-1)$$

$$0 = A + B \qquad \Rightarrow \quad A = 1/2,\ B = -1/2$$

$$1 = A - B$$

$$\int \frac{1}{x^2-1}\,dx = \int \frac{1/2}{x-1} + \frac{-1/2}{x+1}\,dx$$

$$= \ln|x-1| + \ln|x+1| + C$$

6. (15 points) Find

$$\int \frac{x}{x^2+6x+10}\,\mathrm{d}x.$$

$$= \int \frac{\frac{1}{2}(2x+6)-3}{x^2+6x+10}\,dx$$

$$= \frac{1}{2}\int \frac{2x+6}{x^2+6x+10}\,dx - 3\int \frac{1}{(x+3)^2+1}\,dx$$

$$= \frac{1}{2}\ln\left|x^2+6x+10\right| - 3\tan^{-1}(x+3) + C$$