Math 2924-050
Spring 2015
Exam 2

Name: SOLUTIONS

Problem	Points
Problem 1 (20 pts)	
Problem 2 (20 pts)	
Problem 3 (20 pts)	
Problem 4 (20 pts)	
Problem 5 (20 pts)	
Total	

Instructions:

- You are allowed to use a calculator and one 4 inch by 6 inch index card of formulas.
- A list of integral formulas is provided on the last page of the exam.
- You must show your work on any problem that requires a solution of more than one or two lines.
- Some problems are easier than others. Make sure to attempt all the problems before spending a lot of time on the hard ones.

1. (20 points) Find

$$\int x^5 e^{-x^3}\,dx$$

$$= \int x^3 \cdot x^2 e^{-x^3}\,dx = uv - \int v\,du = -\frac{x^3}{3}e^{-x^3} + \int x^2 e^{-x^3}\,dx$$

let $u = x^3$, $dv = x^2 e^{-x^3}\,dx$

$du = 3x^2$, $v = \int x^2 e^{-x^3}\,dx = -\frac{1}{3}e^{-x^3}$

$$= -\frac{x^3}{3}e^{-x^3} - \frac{1}{3}e^{-x^3} + C$$

2. (20 points) Test for convergence/divergence:

$$\sum_{n=1}^{\infty} \frac{n^2 - 2n}{\sqrt{n^4 + 5n^6}}$$

Compare to $\sum \frac{n^2}{\sqrt{n^6}} = \sum \frac{1}{n}$ using limit comp. test:

$$\lim_{n\to\infty} \frac{\frac{n^2-2n}{\sqrt{n^4+5n^6}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3-2n^2}{\sqrt{n^4+5n^6}} = \lim_{n\to\infty} \frac{1-\frac{2}{n}}{\sqrt{\frac{1}{n^2}+5}} = \frac{1}{\sqrt{5}}$$

Since $0 < \frac{1}{\sqrt{5}} < \infty$, the two series are comparable.

$\sum \frac{1}{n}$ div. b/c it is harmonic series. so $\sum \frac{n^2-2n}{\sqrt{n^4+5n^6}}$

div. also.

3. (20 points) Test for convergence/divergence:

$$\sum_{k=1}^{\infty} \frac{k^2 e^k}{k!}$$

Use Ratio Test:

$$\lim_{k\to\infty} \frac{|a_{k+1}|}{|a_k|} = \lim_{k\to\infty} \frac{(k+1)^2 e^{k+1}}{(k+1)!} \cdot \frac{k!}{k^2 e^k}$$

$$= \lim_{k\to\infty} \frac{(k+1)^2}{k^2} \cdot e \cdot \frac{1}{k+1} = 0.$$

So conv. by ratio test.

4. (20 points) Test for convergence/divergence:

$$\sum_{n=2}^{\infty} \frac{\sin(2n)}{(\ln n)2^n}$$

Test for abs. conv. using comp. test:

$$\left| \frac{\sin(2n)}{(\ln n)\cdot 2^n} \right| \leq \frac{1}{(\ln n)\cdot 2^n} \leq \frac{1}{2^n}$$

So $\sum_{n=2}^{\infty} \left| \frac{\sin(2n)}{\ln n \cdot 2^n} \right| \leq \sum_{n=2}^{\infty} \frac{1}{2^n} < \infty$ (geo. series with $r = 1/2$)

So $\sum_{n=2}^{\infty} \left| \frac{\sin(2n)}{\ln n \cdot 2^n} \right|$ conv.

Since abs. conv. $\Rightarrow$ conv., $\sum \frac{\sin(2n)}{(\ln n)\cdot 2^n}$ conv. also

5. (20 points) $\sum_{n=0}^{\infty} a_n$ is a series with the nth partial sum S_n given by $S_n = 7 - \frac{n}{n+1}$. Find the sum of the series

$$\sum_{n=0}^{\infty}\left(a_n + 5\left(-\frac{2}{3}\right)^n\right)$$

$$\sum_{n=0}^{\infty}\left(a_n + 5\left(-\frac{2}{3}\right)^n\right) = \sum_{n=0}^{\infty} a_n + 5\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n$$

$$= \lim_{n\to\infty} S_n + 5\cdot\frac{1}{1-(-2/3)} = 6 + \frac{5}{\frac{5}{3}} = 9$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$

2. $\int \dfrac{1}{x}\,dx = \ln|x|$

3. $\int e^x\,dx = e^x$

4. $\int a^x\,dx = \dfrac{a^x}{\ln a}$

5. $\int \sin x\,dx = -\cos x$

6. $\int \cos x\,dx = \sin x$

7. $\int \sec^2 x\,dx = \tan x$

8. $\int \csc^2 x\,dx = -\cot x$

9. $\int \sec x \tan x\,dx = \sec x$

10. $\int \csc x \cot x\,dx = -\csc x$

11. $\int \sec x\,dx = \ln|\sec x + \tan x|$

12. $\int \csc x\,dx = \ln|\csc x - \cot x|$

13. $\int \tan x\,dx = \ln|\sec x|$

14. $\int \cot x\,dx = \ln|\sin x|$

15. $\int \sinh x\,dx = \cosh x$

16. $\int \cosh x\,dx = \sinh x$

17. $\int \dfrac{dx}{x^2 + a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$

18. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right)$

*19. $\int \dfrac{dx}{x^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right|$

*20. $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$