Review Problems for Exam III

1) Test for conv./div.

a) $$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$$

b) $$\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$$

c) $$\sum_{n=1}^{\infty} \frac{n^{4n}}{(n!)^n}$$

2. Let $f(x) = \sum_{n=0}^{\infty} \frac{1}{(n!)^2} x^n$.

a) Find the interval of convergence.

b) Find $f'(x)$.

c) Find $\lim_{x \to 0} \frac{f(x) - (1 + x + \frac{1}{4}x^2)}{x^3}$

3. Find the Taylor series for e^x centered at $a = -1$.

4. Find the Taylor series (centered at $a = 0$) for the following functions:

a) e^{2x}

b) $x \tan^{-1}\left(\frac{x}{2}\right)$

c) $\frac{x^3}{1+x^2}$

d) $2x \sinh x + \cos 3x$

e) $\dfrac{1}{\sqrt{1+x}}$

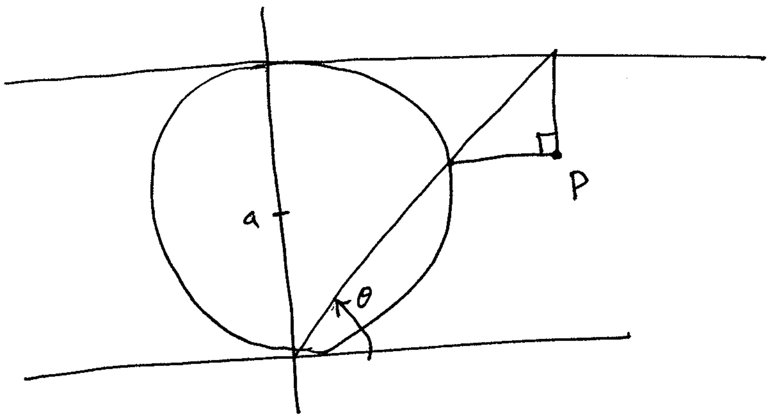
5. Use a power series to evaluate $\int e^{-x^2}\,dx$.

6. It can be shown (using methods of Calc 3) that

$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$. Use this fact and your answer for

#5 to find a series representation of $\sqrt{\pi}$.

7. Find parametric equations, with θ as the parameter, for the curve consisting of all ~~points~~ possible positions of the point P in the figure.



8. Find the arc length of the curve ~~$x = e^t + e^{-t}$, $y = 5 - 2t$~~,

$x = t \sin t$, $y = t \cos t$ $\quad 0 \le t \le 1$.

9. Find the points on the curve where the tangent line is horizontal or vertical. Sketch the curve.

$$x = t^3 - 3t, \; y = t^2 - 3.$$

10. Find the area enclosed by the x-axis and the curve

$$x = 1 + e^t, \quad y = t - t^2.$$

11. Graph the curve $r = 5 \sin \theta$ (in polar coordinates). Find points where tangent line is horizontal or vertical. Find area enclosed by the curve.

12. Find the points of intersection of the curves $r = 2$ and $r = 4\sin(3\theta)$. Find the area of the region outside the circle $r=2$ and inside $r=4\sin(3\theta)$.