Math 2924-050
Fall 2015
Exam 2

SOLUTIONS

Name:_____

Problem	Points
Problem 1 (3 pts)	
Problem 2 (3 pts)	
Problem 3 (3 pts)	
Problem 4 (3 pts)	
Problem 5 (3 pts)	
Total (15 pts)	

Instructions:

- You are allowed to use a calculator and one 4 inch by 6 inch index card of formulas.
- A list of integral formulas is provided on the last page of the exam.
- You must show your work on any problem that requires a solution of more than one or two lines.
- Some problems are easier than others. Make sure to attempt all the problems before spending a lot of time on the hard ones.

1. (3 points) Evaluate the integral

$$\int \frac{\sqrt{2x^2-1}}{x}\,dx$$

$$\int \frac{\sqrt{2x^2-1}}{x}\,dx = \sqrt{2}\int \frac{\sqrt{x^2-1/2}}{x}\,dx = \sqrt{2}\int \frac{\frac{1}{\sqrt{2}}\tan\theta}{\frac{1}{\sqrt{2}}\sec\theta}\,\frac{1}{\sqrt{2}}\sec\theta\tan\theta\,d\theta$$

trig sub $\quad x = \frac{1}{\sqrt{2}}\sec\theta$

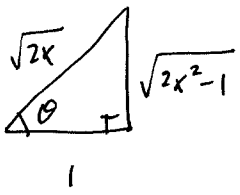
$dx = \frac{1}{\sqrt{2}}\sec\theta\tan\theta\,d\theta$

$$= \int \tan^2\theta\,d\theta$$

$$= \int \left(\sec^2\theta - 1\right) d\theta$$

$$= \tan\theta - \theta + C$$

$$= \sqrt{2x^2-1} - \sec^{-1}\left(\sqrt{2}\,x\right) + C$$



2. (3 points) Evaluate the integral

$$\int \frac{1}{x(x^2+2x+1)}\,dx$$

$$\frac{1}{x(x^2+2x+1)} = \frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$1 = A(x+1)^2 + B(x+1)x + Cx$$

$x=0$: $A = 1$

$x=-1$: $C = -1$

x^2 coeff.: $0 = A + B \implies B = -1$

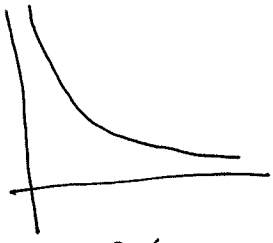
So

$$\int \frac{1}{x(x^2+2x+1)}\,dx = \int \left(\frac{1}{x} + \frac{-1}{x+1} + \frac{-1}{(x+1)^2} \right) dx$$

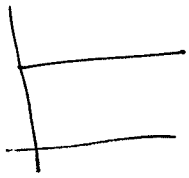
$$= \boxed{\ln|x| - \ln|x+1| + (x+1)^{-1} + C}$$

3. (3 points) For which real numbers p does the following integral converge?

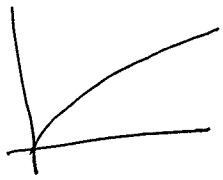
$$\int_0^1 x^p \, dx$$



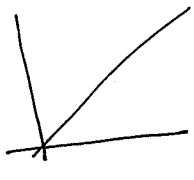
$p < 0$



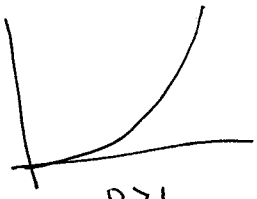
$p = 0$



$0 < p < 1$



$p = 1$



$p > 1$

x^p is cont. on $[0, \infty)$ for $p \geq 0$,

so converges for $p \geq 0$.

For $p < 0$, the integral is improper: For $p \neq -1$:

$$\int_0^1 x^p \, dx = \lim_{h \to 0^+} \int_h^1 x^p \, dx = \lim_{h \to 0^+} \frac{1}{p+1} x^{p+1} \Big|_h^1 = \lim_{h \to 0^+} \left(-\frac{1}{p+1} h^{p+1} + \frac{1}{p+1} \right)$$

$$= \begin{cases} \frac{1}{p+1} & \text{if } p > -1 \\ \infty & \text{if } p < -1 \end{cases}$$

For $p = -1$: $\int_0^1 x^{-1} \, dx = \lim_{h \to 0^+} \ln|x| \Big|_h^1 = \lim_{h \to 0^+} (0 - \ln h) = \infty$

So the integral converges $\iff$ $p > -1$

4. (3 points) Evaluate the inetgral

$$\int \frac{1}{e^x + e^{-x}}\,dx$$

$$\int \frac{1}{e^x + e^{-x}}\,dx = \int \frac{e^x}{e^{2x}+1}\,dx$$

u Sub

let $u = e^x$

$du = e^x\,dx$

$$= \int \frac{1}{u^2+1}\,du$$

$$= \tan^{-1}(u) + C = \boxed{\tan^{-1} e^x + C}$$

5. (3 points) Let a_n be defined recursively by

$$a_1 = 1, \qquad a_n = a_{n-1} + 2^{-n} \quad \text{for } n \geq 2.$$

Does the series

$$\sum_{n=1}^{\infty} a_n$$

converge? Why or why not?

$a_1 = 1$

$a_2 = 1 + 1/4$

$a_3 = 1 + 1/4 + 1/8$

$a_4 = 1 + 1/4 + 1/8 + 1/16$

$\vdots$

$a_n = 1 + 1/4 + 1/8 + 1/16 + \cdots + 2^{-n}$

Thus $a_n > 1$ for all n and $\lim_{n \to \infty} a_n \neq 0$.

Therefore $\sum_{n=1}^{\infty} a_n$ diverges by the test for divergence.

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x} dx = \ln|x|$
3. $\int e^x dx = e^x$
4. $\int a^x dx = \frac{a^x}{\ln a}$
5. $\int \sin x \, dx = -\cos x$
6. $\int \cos x \, dx = \sin x$
7. $\int \sec^2 x \, dx = \tan x$
8. $\int \csc^2 x \, dx = -\cot x$
9. $\int \sec x \tan x \, dx = \sec x$
10. $\int \csc x \cot x \, dx = -\csc x$
11. $\int \sec x \, dx = \ln|\sec x + \tan x|$
12. $\int \csc x \, dx = \ln|\csc x - \cot x|$
13. $\int \tan x \, dx = \ln|\sec x|$
14. $\int \cot x \, dx = \ln|\sin x|$
15. $\int \sinh x \, dx = \cosh x$
16. $\int \cosh x \, dx = \sinh x$
17. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$

*19. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

*20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$