# Math 2924-050
# Fall 2015
# Exam 1

Name: Solutions

| Problem | Points |
|---|---|
| Problem 1 (3 pts) | |
| Problem 2 (3 pts) | |
| Problem 3 (3 pts) | |
| Problem 4 (3 pts) | |
| Problem 5 (3 pts) | |
| Total (15 pts) | |

**Instructions:**

- You are allowed to use a calculator and one 4 inch by 6 inch index card of formulas.
- A list of integral formulas is provided on the last page of the exam.
- You must show your work on any problem that requires a solution of more than one or two lines.
- Some problems are easier than others. Make sure to attempt all the problems before spending a lot of time on the hard ones.

1. (3 points) Find $(f^{-1})'(a)$, where $a = 2$ and $f(x) = \sqrt{x^3 + x^2 + x + 1}$.

$$f(1) = \sqrt{1^3+1^2+1^1+1} = \sqrt{4} = 2$$

So $$f(1) = 2$$
$$1 = f^{-1}(2)$$

Then $$(f^{-1})'(2) = \frac{1}{f'(1)}.$$

$$f'(x) = \frac{3x^2 + 2x + 1}{2\sqrt{x^3 + x^2 + x + 1}}.$$

$$f'(1) = \frac{3+2+1}{2\sqrt{4}} = \frac{3}{2}$$

So $$(f^{-1})'(2) = \frac{1}{f'(1)} \boxed{= \frac{2}{3}}$$

2. (3 points) Evaluate the integral

$$\int \frac{\cosh x}{\cosh^2 x - 1}\, dx$$

$\cosh^2 x - \sinh^2 x = 1$, so

$$\int \frac{\cosh x}{\cosh^2 x - 1}\, dx = \int \frac{\cosh x}{\sinh^2 x}\, dx$$

let $u = \sinh x$

$du = \cosh x \, dx$

$$= \int \frac{1}{u^2}\, du$$

$$= -u^{-1} + C$$

$$\boxed{= -(\sinh x)^{-1} + C}$$

3. (3 points) Find the derivative of $y = x^{\arctan x}$.

$$y = x^{\arctan x} \Rightarrow \ln y = (\arctan x) \cdot (\ln x)$$

$$\Rightarrow \frac{d}{dx}(\ln y) = \frac{d}{dx}\left[(\arctan x) \cdot \ln x\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2} \cdot \ln x + (\arctan x) \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\ln x}{1+x^2} + \frac{\arctan x}{x}\right]$$

$$\boxed{\frac{dy}{dx} = x^{\arctan x} \cdot \left[\frac{\ln x}{1+x^2} + \frac{\arctan x}{x}\right]}$$

4. (3 points) Evaluate the limit

$$\lim_{x \to 0} (\csc x - \cot x)$$

$$\lim_{x \to 0} (\csc x - \cot x) = \lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \left( \frac{1 - \cos x}{\sin x} \right)$$

$\left( \text{Type } \frac{0}{0}, \text{ use L'H.} \right)$ $$= \lim_{x \to 0} \frac{\sin x}{\cos x} = \frac{\sin 0}{\cos 0} = 0$$

5. (3 points) Evaluate the integral

$$\int \cos \sqrt{x}\, dx$$

(Hint: First make a substitution and then integrate by parts.)

let $y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}} dx$

$2y\, dy = dx$

$$\int \cos \sqrt{x}\, dx$$

$$= \int 2y \cos y\, dy$$

$$= 2 \int y \cos y\, dy$$

let $u = y$

$dv = \cos y\, dy$

$du = dy, \quad v = \sin y$

$$= 2 \int u\, dv$$

$$= 2 \left[ uv - \int v\, du \right]$$

$$= 2 \left[ y \sin y - \int \sin y\, dy \right] = 2 \left[ y \sin y + \cos y \right] + C$$

$$= \boxed{2y \sin y + 2 \cos y + C}$$

this is the final answer

$$= \boxed{2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C}$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

**Table of Integration Formulas** Constants of integration have been omitted.

**1.** $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$

**2.** $\int \frac{1}{x}\,dx = \ln|x|$

**3.** $\int e^x\,dx = e^x$

**4.** $\int a^x\,dx = \frac{a^x}{\ln a}$

**5.** $\int \sin x\,dx = -\cos x$

**6.** $\int \cos x\,dx = \sin x$

**7.** $\int \sec^2 x\,dx = \tan x$

**8.** $\int \csc^2 x\,dx = -\cot x$

**9.** $\int \sec x \tan x\,dx = \sec x$

**10.** $\int \csc x \cot x\,dx = -\csc x$

**11.** $\int \sec x\,dx = \ln|\sec x + \tan x|$

**12.** $\int \csc x\,dx = \ln|\csc x - \cot x|$

**13.** $\int \tan x\,dx = \ln|\sec x|$

**14.** $\int \cot x\,dx = \ln|\sin x|$

**15.** $\int \sinh x\,dx = \cosh x$

**16.** $\int \cosh x\,dx = \sinh x$

**17.** $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

**18.** $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$

***19.** $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

***20.** $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$