Math 2924-050
Fall 2014
Exam 3

Name: SOLUTIONS

Problem	Points
Problem 1 (15 pts)	
Problem 2 (15 pts)	
Problem 3 (15 pts)	
Problem 4 (20 pts)	
Problem 5 (15 pts)	
Problem 6 (20 pts)	
Total	

1. (15 points) For which constants $r > 0$ does the integral $\int_0^\infty r^x \, dx$ converge? Evaluate the integral for these values of r.

For $r = 1$, $\int_0^\infty r^x \, dx = \int_0^\infty 1 \, dx$ diverges

For $r > 0$, $r \neq 1$: $\int_0^\infty r^x \, dx = \lim_{N \to \infty} \int_0^N r^x \, dx$

$$= \lim_{N \to \infty} \left. \frac{r^x}{\ln r} \right|_0^N = \lim_{N \to \infty} \left(\frac{r^N}{\ln r} - \frac{1}{\ln r} \right)$$

$$= \begin{cases} -\frac{1}{\ln r}, & 0 < r < 1 \\ \text{diverges}, & r > 1 \end{cases}$$

2. (15 points) Let $\sum_{k=1}^{\infty} a_k$ be a series. Suppose the partial sums S_n are given by the formula

$$S_n = \frac{\sqrt{3n^2+2}}{5n+1}.$$

Find the sum of the series.

$$\sum a_k = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{\sqrt{3n^2+2}}{5n+1}$$

$$= \lim_{n\to\infty} \frac{\sqrt{3+2/n^2}}{5+1/n} = \frac{\sqrt{3}}{5}$$

3. (15 points) Determine if the following series converges:

$$\sum_{n=1}^{\infty} \frac{\ln n}{n}.$$

diverges b/c $\frac{\ln n}{n} \geq \frac{1}{n}$ and $\sum \frac{1}{n}$ div.

4. (20 points) For which $p > 0$ does the following series converge?

$$\sum_{n=1}^{\infty} \sin(1/n^p)$$

Compare to $\sum \frac{1}{n^p}$

$$\lim_{n \to \infty} \frac{\sin(1/n^p)}{1/n^p} = \lim_{u \to 0} \frac{\sin(u)}{u} = 1$$

$$(u = 1/n^p)$$

So $\sum \sin(1/n^p)$ conv. $\Leftrightarrow$ $p > 1$

5. (15 points) Determine if the following series converges:

$$\sum_{n=1}^{\infty}(-1)^n \frac{n^n}{n!}.$$

div. because $\lim_{n\to\infty} \frac{n^n}{n!} = \infty \quad (\neq 0)$

6. (20 points) Determine if the following series converges:

$$\sum_{n=1}^{\infty} \frac{n^{100} 2^n}{1+3^n}.$$

$$\sum \frac{n^{100} 2^n}{1+3^n} \leq \sum \frac{n^{100} 2^n}{3^n} < \infty$$

by ratio test: $\lim_{n\to\infty} \frac{(n+1)^{100} 2^{n+1}}{3^{n+1}} \cdot \frac{3^n}{n^{100} 2^n} = \frac{2}{3} < 1$