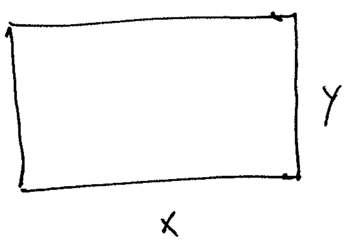
Problem 1

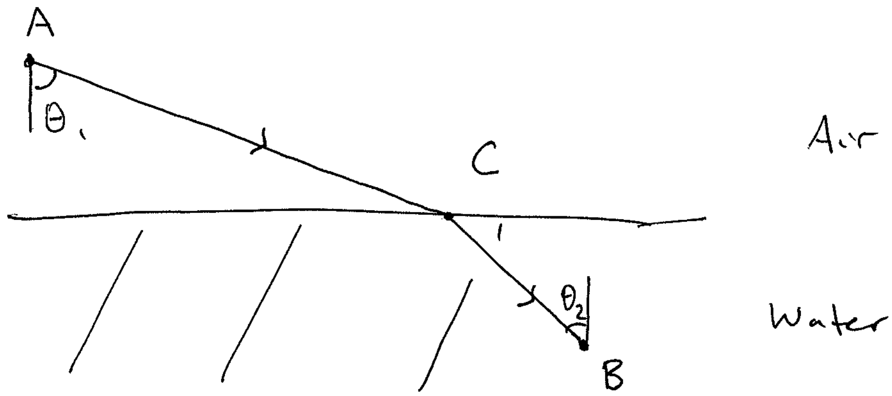
Show that among all rectangles with a fixed perimeter P, the square with sides of length $\frac{P}{4}$ maximizes area.



Constraint: $P = 2x + 2y$

Maximize: $A = xy$

Problem 2



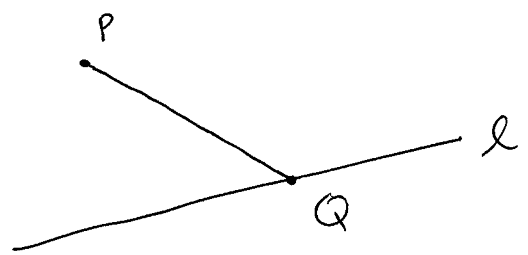
A light ray refracts (changes its path) when it passes from one medium to another, for example from air to water.

This can be explained by the following principle:

The speed of light in air is v_1, the speed of light in water is v_2, and the path from A to B that the light ray takes is the path that minimizes the time taken.

Show that $$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}.$$

Problem 3



Given a line ℓ and a point P not on ℓ, let $d = d(P, Q)$ be the distance between P and Q. Show that d is minimized when $\overline{PQ} \perp \ell$. (P is fixed and Q can vary on ℓ.)

Hints:

1) d is minimized $\Leftrightarrow$ d^2 is minimized

2) lines with slopes m_1 and m_2 are perpendicular $\Leftrightarrow$ $m_1 m_2 = -1$

3) Without loss of generality you may assume $P = (0,0)$ and the eqtn of ℓ is $y = mx + b$ with $b \neq 0$.

Problem 4

Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.