

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Find $\lim_{x \rightarrow \infty} (\sqrt{9x^4 + x} - 3x^2)$.

$$\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{9x^4 + x} - 3x^2) &= \lim_{x \rightarrow \infty} \frac{\sqrt{9x^4 + x} - 3x^2}{1} \cdot \frac{\sqrt{9x^4 + x} + 3x^2}{\sqrt{9x^4 + x} + 3x^2} = \lim_{x \rightarrow \infty} \frac{9x^4 + x - 9x^4}{\sqrt{9x^4 + x} + 3x^2} \\ &= \lim_{x \rightarrow \infty} \frac{x}{\sqrt{9x^4 + x} + 3x^2} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{9x^2 + \frac{1}{x}} + 3x} = 0 \end{aligned}$$

Problem 2: (3 points) Find $\lim_{x \rightarrow \infty} x^2 \sin(1/x)$.

$$\lim_{x \rightarrow \infty} x^2 \sin\left(\frac{1}{x}\right) = \lim_{x \rightarrow \infty} x \cdot \left[x \sin\left(\frac{1}{x}\right) \right] = \infty \quad \text{because}$$

$$\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) = \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{u \rightarrow 0} \frac{\sin(u)}{u} = 1$$

Problem 3: (4 points) Find the vertical and horizontal asymptotes of the function

$$f(x) = \frac{2x^2 - 5}{x^2 - 4}$$

H.A.: $\lim_{x \rightarrow \infty} \frac{2x^2 - 5}{x^2 - 4} = 2$, so $y = 2$ is H.A.

V.A.: $x^2 - 4 = 0 \iff x = \pm 2$, so $x = 2, x = -2$ are V.A.s