

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (5 points) Suppose $y = y(x)$ is a function of x and satisfies

$$(1+x)y' + y^2 = 3 \cos x, \quad y(0) = 1.$$

Find $y'(0)$ and $y''(0)$.

$$(1+0)y'(0) + y(0)^2 = 3 \cos 0$$

$$y'(0) + 1 = 3$$

$$y'(0) = 2$$

By implicit diff.,

$$(1+x)y'' + y' + 2yy' = -3 \sin x$$

Plug in $x=0$:

$$y''(0) + y'(0) + 2y(0)y'(0) = -3 \sin 0$$

$$y''(0) + 2 + 2 \cdot 1 \cdot 2 = 0$$

$$y''(0) = -6$$

Problem 2: (5 points) The equation of a circle of radius 5 is $x^2 + y^2 = 25$. Find the equation of the tangent line at the point $(3, 4)$.

$$2x + 2yy' = 0$$

$$y' = -x/y$$

$$\text{At } (3, 4) : y' = -3/4$$

$$\text{So eqn of tangent line is } y - 4 = -3/4(x - 3)$$

$$y = -3/4x + 25/4$$