Calculus I, Fall 2014
Quiz 3 Name: _____

You must show all your work to receive credit. Calculators are allowed.

Problem 1: (3 points) Find the exact value of

$$\lim_{t\to 2}\sqrt{\frac{t^2-t-2}{t-2}}$$

$$\sqrt{\frac{t^2-t-2}{t-2}} = \sqrt{\frac{(t-2)(t+1)}{t-2}} = \sqrt{t+1}, \text{ so}$$
$$\lim_{t\to 2}\sqrt{\frac{t^2-t-2}{t-2}} = \lim_{t\to 2}\sqrt{t+1} = \sqrt{3}.$$

Problem 2: (3 points) Explain what is wrong with the following reasoning:

$$\lim_{x\to 0}[x\sin(1/x)] = \left[\lim_{x\to 0} x\right]\cdot\left[\lim_{x\to 0}\sin(1/x)\right] = 0\cdot\left[\lim_{x\to 0}\sin(1/x)\right] = 0.$$

The first step is incorrect because the limit product law cannot be used due to the fact that $\lim_{x\to 0}\sin(1/x)$ does not exist.

Problem 3: (4 points) Using the ϵ, δ definition of a limit, prove that

$$\lim_{x\to 3}(2x+1) = 7.$$

Make sure you separate your scratch work from your formal proof. Use the back of the quiz if you need more room.

Let $\epsilon > 0$ be given. Choose $\delta = \epsilon/2$. Then $|f(x)-L| = |2x+1-7| = |2x-6| = 2|x-3| < 2\delta = \epsilon$ whenever $0 < |x-3| < \delta$.