Calculus and Analytic Geometry 1, Math 1823-001, Fall 2014
Practice Exam 1

1. Find the following limits exactly. Write DNE if they do not exist. Allow $\infty, -\infty$ as possible answers.

   a) $\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right)$

   b) $\lim_{h\to 3}\frac{h^2+2}{h+3}$

   c) $\lim_{x\to -3}\left(\frac{x^2-9}{x+3}\right)^{1/3}$

   d) $\lim_{y\to -1}(\cos(y+1)\sin(\pi y/2))$

   e) $\lim_{x\to 0}\sin^2(1/x)$

   f) $\lim_{x\to 1}\frac{1/x-1}{x-1}$

   g) $\lim_{x\to\sqrt{2}}(2x^2-3x+5)$

   h) $\lim_{x\to 1^-}\frac{2(x-1)}{|x-1|}$

   i) $\lim_{t\to 0^+}\frac{1}{\sin t}$

2. a) Explain with a picture why

$$\lim_{x \to 0} [x \sin(1/x)] = 0.$$

b) Let

$$f(x) = \begin{cases} x \sin(1/x) & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Explain why $f'(0)$ does not exist.

c) Let

$$g(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Show that $g'(0) = 0$.

3. Let $f : [-1, 1] \to \mathbb{R}$ be the function $f(x) = \sqrt{1 - x^2}$. The graph of f is the upper half of a circle of radius 1. Draw the graph and use it to explain why

$$\lim_{a \to -1^+} f'(a) = \infty, \qquad \lim_{a \to 1^-} f'(a) = -\infty.$$

4. Explain the geometric significance of each of the following limits:

$$\lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h}, \qquad \lim_{x \to a} \frac{x^2 - a^2}{x - a}.$$

5. Draw a picture of a function that is continuous everywhere except at $x = -2$ and $x = 2$, and, furthermore, is continuous from the left at -2 and not continuous from the right or left at 2.

6. Let

$$f(x) = \begin{cases} 2x^2 + ax & x \leq 1, \\ -3x + 2 & x > 1. \end{cases}$$

Find the number a so that f is continuous everywhere.

7. Let $P_1(x)$ and $P_2(x)$ be polynomials. Let $Q(x) = P_1(x)/P_2(x)$. At which points is $Q(x)$ discontinuous?

8. In the ϵ, δ definition of a limit, the inequalities

$$|f(x) - L| < \epsilon, \qquad 0 < |x - a| < \delta$$

appear. What are the geometric meanings of these inequalities?

9. a) Let $f(x) = -x^2 + 1$. For a an arbitrary number, find $f'(a)$.

b) Find the equation of the line tangent to the graph of f at $x = 3$.

10. Let $h(t) = \cos t$. Draw the graph of h, and use the graph to find $h'(0)$.

11. Suppose $f(x)$ and $g(x)$ are continuous functions on all of $\mathbb{R}$, and

$$f(2) = 4, \quad g(2) = 3, \quad g(1) = 2.$$

Find the following limits:

a) $\lim_{x \to 2} [2f(x) - g(x)]$

b) $\lim_{x \to 1} (f \circ g)(x)$

c) $\lim_{x \to 2} \sqrt{f(x)}$

d) $\lim_{x \to 2} \sin\left(\dfrac{f(x)}{g(x)}\right)$