Math 1823-001
Fall 2014
Exam 3

Name: SOLUTIONS

Problem	Points
Problem 1 (15 pts)	
Problem 2 (15 pts)	
Problem 3 (10 pts)	
Problem 4 (20 pts)	
Problem 5 (15 pts)	
Problem 6 (10 pts)	
Problem 7 (15 pts)	
Total	

1. (15 points) If $xy + y^3 = 8$, find the value of y' when $x = 0$.

1) $X = 0 \Rightarrow 0 \cdot y + y^3 = 8$

$$y^3 = 8$$

$$y = 2$$

So $y = 2$ when $x = 0$

2) $$(xy + y^3)' = (8)'$$

$$y + xy' + 3y^2 y' = 0$$

$$y'(x + 3y^2) = -y$$

$$y' = -\frac{y}{(x + 3y^2)}$$

So when $X = 0$,

$$y' = -\frac{2}{(0 + 3 \cdot 2^2)} = -\frac{\cancel{2}1}{3 \cdot 2} = -1/6$$

2. (15 points) A particle is moving along the curve defined by $xy = 2$. As it reaches the point $(1, 2)$, the y-coordinate is decreasing by 2 m/sec. How fast is the x-coordinate of the particle changing at this instant?

$$\frac{d}{dt}(xy) = \frac{d}{dt}(2)$$

$$\frac{dx}{dt} y + x \frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{x}{y} \frac{dy}{dt}$$

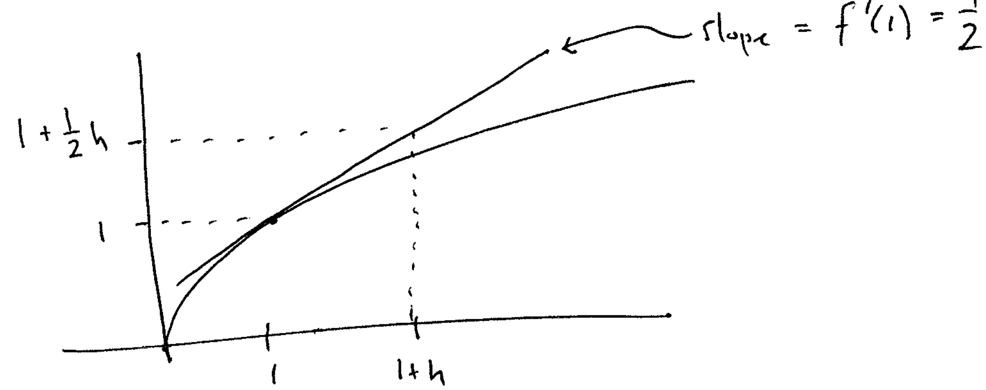
when $(x, y) = (1, 2)$, $\frac{dy}{dt} = -2$ m/sec

so $\frac{dx}{dt} = -\frac{1}{2}(-2) = 1$ m/sec

3. (10 points) Suppose h is a small number. Use linear approximation to approximate $\sqrt{1+h}$.

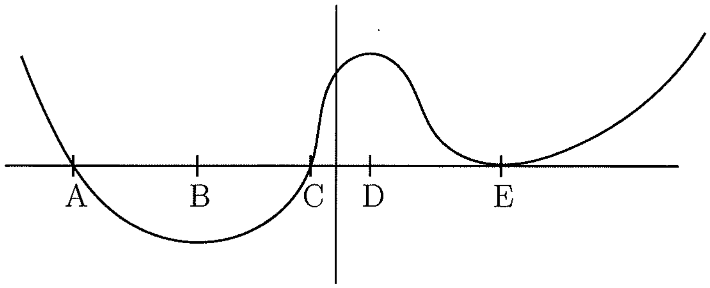
let $f(x) = \sqrt{x}$, $f(1) = 1$

$f'(x) = \dfrac{1}{2\sqrt{x}}$, $f'(1) = \dfrac{1}{2}$



so $\sqrt{1+h} \approx 1 + \frac{1}{2}h$

4. (20 points) The graph of f' is given below.



a) What are the critical numbers of f?

$f' = 0$ at A, C, E

b) What are the intervals over which f is decreasing? What about increasing?

f inc. $\Longleftrightarrow$ $f' \geq 0$ $\Longleftrightarrow$ intervals $(-\infty, A)$, (C, E) (or (C, E), (E, ∞) is ok too)

f dec $\Longleftrightarrow$ $f' \leq 0$ $\Longleftrightarrow$ interval (A, C)

c) Where does f have local maximums? What about local minimums?

local max at A by 1^{st} der. test

local min at C by 1^{st} der test

d) What are the intervals over which f is concave up? What about concave down?

f concave up $\Longleftrightarrow$ f' inc. $\Longleftrightarrow$ intervals (B, D), (E, ∞)

f concave down $\Longleftrightarrow$ f' dec $\Longleftrightarrow$ intervals $(-\infty, B)$, (D, E)

e) Where are the inflection points of f?

f infl. pts. $\Longleftrightarrow$ f' local max/min $\Longleftrightarrow$ B and D

5. (15 points) Find the absolute maximum value and the absolute minimum value of $f(x) = x^3 - x$ on the interval $[-2, 1]$.

$$f'(x) = 3x^2 - 1 = 0$$

$$x^2 = \frac{1}{3}$$

$x = \pm \frac{1}{\sqrt{3}}$ are crit. #s.

$$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}} = -\frac{2}{3\sqrt{3}} \approx -.38$$

$$f\left(-\frac{1}{\sqrt{3}}\right) = -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} \approx .38$$

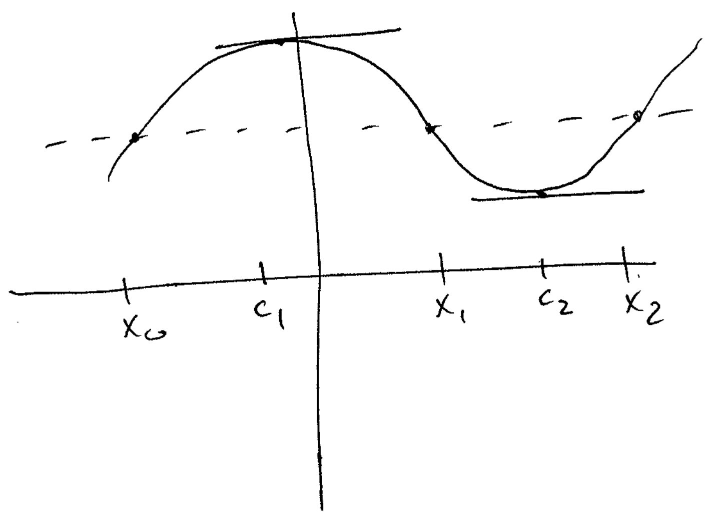
end pts : $f(1) = 1 - 1 = 0$

$f(-2) = -8 + 2 = -6$

abs. max at $-\frac{1}{\sqrt{3}}$ is $f\left(-\frac{1}{\sqrt{3}}\right) = \frac{2}{3\sqrt{3}}$

abs min at -2 is $f(-2) = -6$

6. (10 points) Suppose f is a differentiable function such that there are three numbers x_0, x_1, x_2 with $x_0 < x_1 < x_2$ and $f(x_0) = f(x_1) = f(x_2)$. Prove that f has at least two critical numbers.



by Rolle's Thm / MVT, there exists c_1 between x_0 and x_1 with $f'(c_1) = 0$

Likewise there exists c_2 between x_1 and x_2 with $f'(c_2) = 0$.

Thus c_1 and c_2 are crit. numbers for f. So f has at least 2 crit. numbers.

7. (15 points) A spherical balloon is inflated at a rate of 8 cm^3/sec. Find the rate of change of the surface area of the balloon when the radius is 2 cm. (The formula for the volume of a sphere is $V = 4\pi r^3/3$, and the formula for the surface area of a sphere is $A = 4\pi r^2$.)

Given $\dfrac{dV}{dt} = 8 \ \text{cm}^3/\text{sec}$

Find $\dfrac{dA}{dt}$ when $r = 2$ cm

$V = \frac{4}{3}\pi r^3, \quad \dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}, \quad$ so $\dfrac{dr}{dt} = \dfrac{dV/dt}{4\pi r^2}$

$A = 4\pi r^2, \quad \dfrac{dA}{dt} = 8\pi r \dfrac{dr}{dt} = 8\pi r \cdot \dfrac{dV/dt}{4\pi r^2}$

$= \dfrac{2 \ dV/dt}{r}$

So when $r = 2$, $\dfrac{dA}{dt} = \dfrac{2 \cdot 8}{2} = 8 \ \text{cm}^2/\text{sec}$