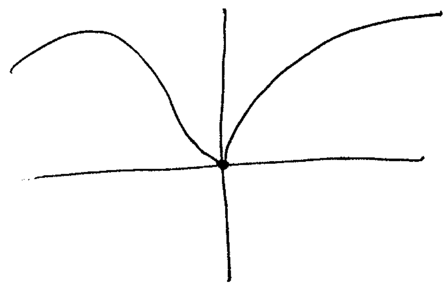
Math 1823-001
Fall 2014
Exam 2

Name: SOLUTIONS

Problem	Points
Problem 1 (10 pts)	
Problem 2 (5 pts)	
Problem 3 (25 pts)	
Problem 4 (10 pts)	
Problem 5 (10 pts)	
Problem 6 (10 pts)	
Problem 7 (10 pts)	
Problem 8 (10 pts)	
Problem 9 (10 pts)	
Total	

1. (10 points) Is it possible for a function to be continuous at 0 but not differentiable at 0? If so, draw an example of such a function; if not, write impossible.

yes:



Is it possible for a function to be differentiable at 0 but not continuous at 0? If so, draw an example of such a function; if not, write impossible.

impossible

2. (5 points) Use the limit definition of the derivative to find the derivative of $f(x) = x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} (2x + h) = 2x$$

3. (25 points) Find the derivatives of the following functions.

a) $-3x^2 + \sqrt{x} + 2x - 10$

$$-6x + \frac{1}{2} x^{-1/2} + 2$$

b) $(\sin x + 1)^{-10}$

$$-10(\sin x + 1)^{-11} \cdot \cos x$$

c) $\frac{2x^2-3x}{1-5x}$

$$\frac{(4x-3)(1-5x) - (2x^2-3x)(-5)}{(1-5x)^2}$$

d) $\sec\left(2 + \frac{1}{1+x}\right)$

$$\sec\left(2+\frac{1}{1+x}\right)\tan\left(2+\frac{1}{1+x}\right)\cdot\left(-(1+x)^{-2}\right)$$

e) $(1 + x^\pi)(4x + \sqrt{1-x})^3$.

$$(\pi x^{\pi-1})\left(4x+\sqrt{1-x}\right)^3 + (1+x^\pi)\,3\left(4x+\sqrt{1-x}\right)^2\left(4-(1-x)^{-1/2}\right)$$

4. (10 points) Let $f(x) = 3 + \cos^2(x+1)$. Find the rate of change of the rate of change of f at $x = 1$.

$$f'(x) = 2\cos(x+1) \cdot (-\sin(x+1))$$

$$= -2\cos(x+1)\sin(x+1)$$

$$f''(x) = 2\sin^2(x+1) - 2\cos^2(x+1)$$

$$f''(1) = 2\sin^2(2) - 2\cos^2(2)$$

5. (10 points) Find the derivative of $\sqrt{x + \sqrt{x + \sqrt{x}}}$.

$$\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} \cdot \left[1 + \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right)\right]$$

6. (10 points) Find

$$\lim_{t\to 0} \frac{\sin^2(3t)}{t^2}.$$

$$= \lim_{t\to 0} \frac{\sin(3t)}{t} \cdot \frac{\sin(3t)}{t} = \lim_{t\to 0}\left(9 \cdot \frac{\sin(3t)}{3t} \cdot \frac{\sin(3t)}{t}\right)$$

$$= 9\left(\lim_{t\to 0} \frac{\sin(3t)}{3t}\right) \cdot \left(\lim_{t\to 0} \frac{\sin(3t)}{3t}\right) = 9 \cdot 1 \cdot 1 = 9$$

7. (10 points) Let $A(t) = f(t)g(t)$. Show that

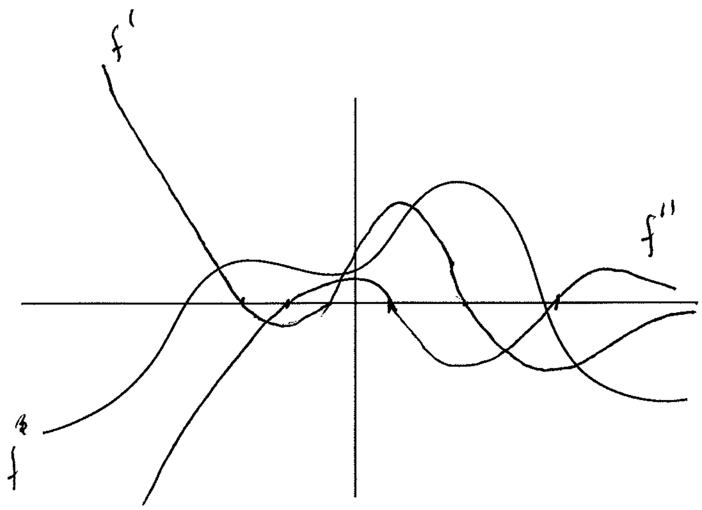
$$A''(t) = f''(t)g(t) + 2f'(t)g'(t) + f(t)g''(t).$$

$$A'(t) = f'(t)\,g(t) + f(t)\,g'(t)$$

$$A''(t) = f''(t)g(t) + f'(t)g'(t) + f'(t)g'(t) + f(t)g''(t)$$

$$= f''(t)g(t) + 2f'(t)g'(t) + f(t)g''(t)$$

8. (10 points) The graph of a function $f(x)$ is given below. Draw in the graphs of $f'(x)$ and $f''(x)$. Make sure to label which graph is which.



9. (10 points) Let $f(x) = x^2 + Ax + B$ with A and B constants. Suppose the tangent line is horizontal at $x = 2$ and $f(0) = 5$. Find A and B.

$f(0) = 5 \Rightarrow B = 5$

$0 = f'(2) = 2 \cdot 2 + A \Rightarrow A = -4$