Math 1823-001
Fall 2014
Exam 1

Name:____

Problem	**Points**
Problem 1 (20 pts)	
Problem 2 (10 pts)	
Problem 3 (15 pts)	
Problem 4 (10 pts)	
Problem 5 (10 pts)	
Problem 6 (10 pts)	
Problem 7 (15 pts)	
Problem 8 (10 pts)	
Total	

1. (20 points) Find the following limits exactly. Write DNE if they do not exist. Allow $\infty, -\infty$ as possible answers.

a) $\lim_{x\to -1} \sin(x^3+2x^2-3x+4)$ $= \sin(-1+2+3+4)$

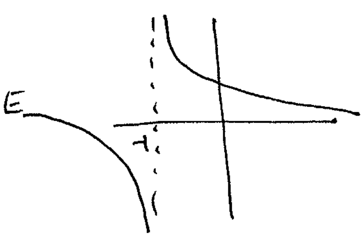
$= \sin 8$

b) $\lim_{h\to 0} \dfrac{\frac{1}{h+1}-1}{h}$ $= \lim_{h\to 0}\left[\dfrac{1}{h(h+1)} - \dfrac{1}{h}\right] = \lim_{h\to 0}\left[\dfrac{1}{h(h+1)} - \dfrac{h+1}{h(h+1)}\right]$

$= \lim_{h\to 0} \dfrac{-\cancel{h}}{\cancel{h}(h+1)} = \dfrac{-1}{1} = -1$

c) $\lim_{x\to 2} \sqrt{\dfrac{x^2-4}{x-2}}$ $= \lim_{x\to 2} \sqrt{\dfrac{(x-2)(x+2)}{x-2}} = \lim_{x\to 2} \sqrt{x+2} = \sqrt{2+2} = 2$

f) $\lim_{x\to -1} \dfrac{x+2}{x+1}$ $=$ DNE



e) $\lim_{x\to 5^+} f(x)$, where $f(x) = \begin{cases} (x-5)^2, & x \le 5 \\ 1/(x-5), & x > 5 \end{cases}$

$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} \dfrac{1}{x-5} = \infty$

2. a) (5 points) Let $f(x)$ be a function and a be a number. Write down the definition of $f'(a)$ as a limit. Geometrically, what is the significance of $f'(a)$?

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f'(a)$ = slope of tangent line at $x = a$

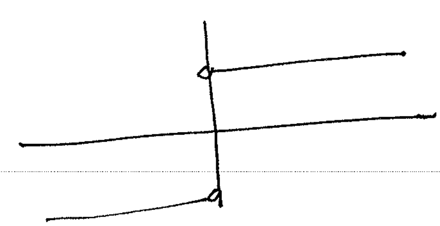
b) (5 points) Let $f(x) = |x|$. Using the limit definition of a derivative, show that $f'(0)$ does not exist.

$$f'(0) = \lim_{h \to 0} \frac{|0+h| - |0|}{h} = \lim_{h \to 0} \frac{|h|}{h}$$

DNE

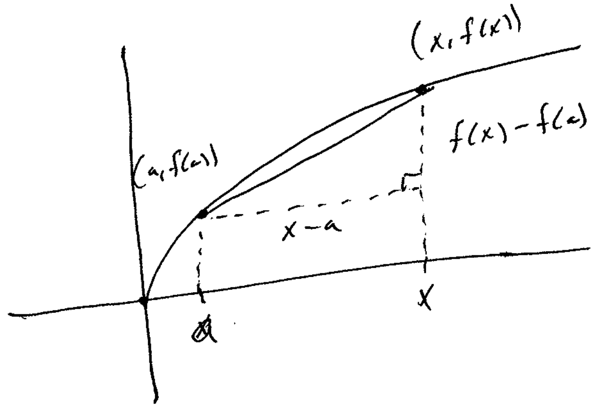
(because left/right limits are not equal)

$\frac{|h|}{h}$



3. Let $f(x) = \sqrt{x}$.

a) (5 points) Write down a formula for the slope of the secant line between the points $(a, f(a))$ and $(x, f(x))$ on the graph of f. Draw a picture to illustrate your answer.



$$\text{slope} = \frac{f(x) - f(a)}{x - a} = \frac{\sqrt{x} - \sqrt{a}}{x - a}$$

b) (5 points) Find the slope of the line tangent to the graph of f at a by taking the limit as $x \to a$ of your answer from part a).

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} = \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}$$

$$= \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$

c) (5 points) Find the equation of the line tangent to the graph of f at $a = 4$.

$$\text{Slope} = \frac{1}{2\sqrt{a}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}, \quad \text{point} = (4, f(4)) = (4, 2)$$

$$\Rightarrow \quad y - 2 = \frac{1}{4}(x - 4) \Rightarrow y = \frac{1}{4}x + 1$$

4. (10 points) Suppose $\lim_{x\to a} f(x)$ exists and $\lim_{x\to a} g(x)$ exists. Is it always true that

$$\lim_{x\to a} \frac{f(x)}{g(x)}$$

exists? Why or why not?

No, for example if $g(x) = 0$ for all x the limit does not exist.

5. (10 points) Suppose $g(t)$ and $h(t)$ are continuous functions, and

$$\lim_{t\to a} g(t) = L_1, \qquad \lim_{t\to a} h(t) = L_2.$$

Find $\lim_{t\to a}[g(t) + 2h(t)]^2$.

$$\lim_{t\to a} [g(t) + 2h(t)]^2 = \left[\lim_{t\to a} g(t) + 2\lim_{t\to a} h(t)\right]^2$$

$$= [L_1 + 2L_2]^2$$

6. The ϵ, δ definition of a limit is the following:

$\lim_{x \to a} f(x) = L$ if, for every $\epsilon > 0$, there is some $\delta > 0$ so that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.

a) (5 points) Why does the definition say $0 < |x - a| < \delta$ instead of just $|x - a| < \delta$?

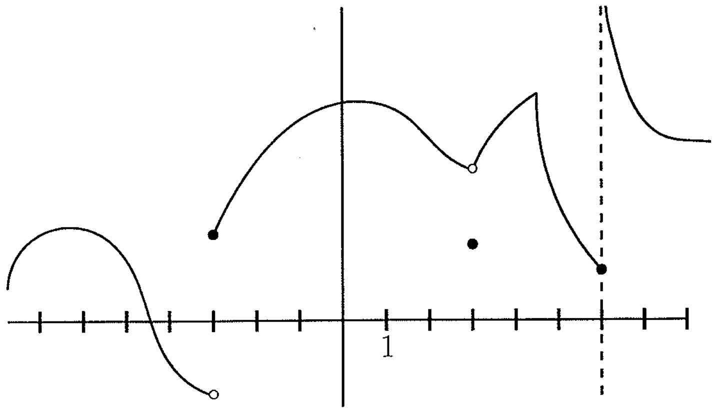
Because we want X to not be equal to a.

b) (5 points) For an infinite limit, the definition is slightly different. It is: $\lim_{x \to a} f(x) = \infty$ if, for every $M > 0$, there is some $\delta > 0$ so that $f(x) > M$ whenever $0 < |x - a| < \delta$.

The other definition does not make sense for an infinite limit because it does not make sense to say $|f(x) - \infty| < \epsilon$. Explain (in words) why $|f(x) - \infty| < \epsilon$ does not make sense.

$|f(x) - \infty|$ is not defined; geometrically, $|f(x) - \infty|$ would be the distance between $f(x)$ and ∞, which would always be ∞ no matter what $f(x)$ is.

7. (15 points) Here is the graph of a function f.



Use the graph to answer the following questions.

a) At which numbers is the function discontinuous?

$-3, 3, 6$

b) For each number from the previous part, state whether or not $\lim_{x \to a} f(x)$ exists (a stands for the number).

$\lim_{x \to -3} f(x)$ DNE

$\lim_{x \to 6} f(x)$ DNE

$\lim_{x \to 3} f(x)$ exists

8. (10 points) Let $f(x) = x^3 + x^2 + x + 1$. Use the intermediate value theorem (IVT) to show that there is a number c between 0 and 2 such that $f(c) = 10$.

$f(0) = 1 < 10 < 15 = f(2)$, and

f is continuous.

Therefore the IVT says there is a number c between 0 and 2 so that $f(c) = 10$.