1. (a) If \( r: X \to A \) is a retraction, what can you say about the map \( i_*: \pi_1(A, a_0) \to \pi_1(X, a_0) \), where \( i: A \hookrightarrow X \) is inclusion and \( a_0 \in A \)? Give a proof of your statement.

(b) Show that the fundamental group of the “figure eight” is infinite, by using a retraction to a subspace. [The figure eight is the union of two circles that touch in one point.]

2. (a) State the Tietze extension theorem.

(b) Let \( Z \) be a space which is a union of two closed sets \( X \cup Y \), where \( X \) and \( Y \) are normal. Show that \( Z \) is normal.

3. Let \( X_0 \) be a path component of \( X \) and let \( x_0 \in X_0 \) be a basepoint. Show that the inclusion map \( X_0 \hookrightarrow X \) induces an isomorphism of fundamental groups \( \pi_1(X_0, x_0) \to \pi_1(X, x_0) \).

4. (a) State the Borsuk-Ulam theorem for \( S^2 \).

(b) Suppose that the sphere \( S^2 \) is expressed as a union of three closed sets: \( S^2 = A_1 \cup A_2 \cup A_3 \). Show that one of the sets \( A_i \) contains an antipodal pair \( \{x, -x\} \). [Hint: use the functions \( f_i(x) = \text{dist}(x, A_i) \) for \( i = 1, 2 \).]

5. Let \( p: E \to B \) be a covering map.

(a) Show that if \( B \) is Hausdorff then so is \( E \).

(b) Suppose \( p(e_0) = b_0 \). Show that the induced homomorphism \( p_*: \pi_1(E, e_0) \to \pi_1(B, b_0) \) is injective. State clearly any theorems that you use.

6. (a) Show that if \( Y \) is Hausdorff then the space of continuous maps \( C(X, Y) \) with the compact-open topology is Hausdorff.

(b) Consider the sequence of functions \( f_n \in C(\mathbb{R}, \mathbb{R}) \) given by \( f_n(x) = x/n \). Does this sequence converge in the compact-open topology? Explain why or why not.