1. Let $X$ be the quotient space obtained from $\mathbb{C} \times \{0, 1\}$ by identifying $z \times 0$ with $z \times 1$ for every complex number $z$ with $|z| > 1$.
   (a) Does $X$ satisfy the $T_1$ axiom? Why or why not?
   (b) Is $X$ Hausdorff? Why or why not?

2. Prove the Tube Lemma: Consider the product space $X \times Y$ where $Y$ is compact. If $N$ is an open set of $X \times Y$ containing the subset $x_0 \times Y$, then $x_0$ has a neighborhood $W$ in $X$ such that $W \times Y$ is contained in $N$.

3. Recall that a space $X$ is locally connected if there is a basis for the topology of $X$ consisting of connected sets.
   (a) Give an example of a space that is locally connected but not connected (no proof required).
   (b) Give an example of a space that is connected but not locally connected (no proof required).
   (c) Prove that $X$ is locally connected if and only if for every open set $U$ of $X$, each component of $U$ is open in $X$.

4(a) Define the finite intersection property and state (without proof) a definition of compactness in terms of this property.
   (b) Prove the contraction mapping theorem: Let $f: X \to X$ be a continuous map where $X$ is a compact metric space, and suppose there is a number $\lambda < 1$ such that
   $$d(f(x), f(y)) \leq \lambda d(x, y)$$
   for all $x, y \in X$. Then there is a unique point $x \in X$ such that $f(x) = x$.

5. Consider the following subspace $D$ of the plane. Let $K = \{1/n \mid n \in \mathbb{Z}_+\}$ and define
   $$D = ([0, 1] \times 0) \cup (K \times [0, 1]) \cup (0 \times 1).$$
   Let $p \in D$ be the point $0 \times 1$.
   (a) Is $D$ connected? Why or why not?
   (b) Show that $D$ is not path connected, as follows. Consider any path $f: [0, 1] \to D$ with $f(0) = p$. Show that $f(1)$ must also be $p$, by showing that $f^{-1}\{p\} = [0, 1]$. [Hint: show that $f^{-1}\{p\}$ is open in $[0, 1]$. Then show that it is closed.]

6. Let $p: X \to Y$ be a quotient map. Show that if each set $p^{-1}\{y\}$ is connected and $Y$ is connected, then $X$ is connected.