1. Let $X$ be the set of real numbers with the finite complement topology (complements of finite sets are open).

(a) Does $X$ satisfy the $T_1$ axiom?
(b) Is $X$ Hausdorff?
(c) To what point or points does the sequence $x_n = 1/n$ converge?

2. Consider the space $X = \mathbb{Z}^+ \times [0,1)$ in the dictionary order topology. (The sets $\mathbb{Z}^+$ and $[0,1)$ are given their usual orderings.) Construct a homeomorphism from $X$ to the subspace $[0, \infty) \subset \mathbb{R}$ (and show that it is a homeomorphism).

3. Let $f: A \to X \times Y$ be given by $f(a) = (f_1(a), f_2(a))$ where $f_1: A \to X$ and $f_2: A \to Y$ are functions. Show that if $f_1$ and $f_2$ are continuous then so is $f$.

4. Let $f: \mathbb{R} \to \mathbb{R}^\omega$ be given by $f(t) = (t, \frac{1}{2}t, \frac{1}{4}t, \frac{1}{8}t, \ldots )$.

(a) Show that $f$ is continuous if $\mathbb{R}^\omega$ is given the product topology.
(b) Show that $f$ is not continuous if $\mathbb{R}^\omega$ is given the box topology.

5. Determine the closures of the following sets:

(a) $A = \{(1/n) \times 0 \mid n \in \mathbb{Z}^+\}$ in the ordered square ($I \times I$ in the dictionary topology)
(b) $K = \{1/n \mid n \in \mathbb{Z}^+\}$ in the set $\mathbb{R}$ with topology given by the basis $\{(-\infty, a) \mid a \in \mathbb{R}\}$
(c) $B = \{x \times \frac{1}{x} \mid 0 < x < 1\}$ in the ordered square

6. Let $a, b$ be points in a space $Z$. A path from $a$ to $b$ is a continuous map $f: [0,1] \to Z$ such that $f(0) = a$ and $f(1) = b$. Consider the following subspace of $\mathbb{R}^2$:

$$Z = \{(x, y) \mid x \in \mathbb{Q}, \ y > 0\} \cup \{(x, 0) \mid x \in \mathbb{R}\}.$$ 

Let $a = (x_a, y_a) \in Z$ and $b = (x_b, y_b) \in Z$, where $x_a < x_b$.

(a) Show that for any path in $Z$ from $a$ to $b$, its image contains the interval $(x_a, x_b) \times \{0\}$.
(b) Given arbitrary points $a, b \in Z$, describe the shortest path in $Z$ from $a$ to $b$, and write down an expression for its length. (This defines a metric on $Z$, different from the usual metric in $\mathbb{R}^2$.)