Exam I
Math 2513-001
February 16, 2009

Problem 1: 
Problem 2: 
Problem 3: 
Problem 4: 
Total: 
1. Use a truth table to show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \lor r)$ are logically equivalent. Don’t just give the table; say briefly why the table shows this.
2(a) Let $S(x)$ be the predicate “$x$ is a student,” $F(x)$ the predicate “$x$ is a faculty member,” and $A(x, y)$ the predicate “$x$ has asked $y$ a question,” where the domain consists of all people associated with OU. Use quantifiers to express these statements:

(i) Every student has asked Professor Gross a question.
(ii) There is a faculty member who has never been asked a question by a student.

2(b) Let $T(x, y)$ mean that $x$ likes cuisine $y$, where the domain for $x$ consists of all students at OU and the domain for $y$ consists of all cuisines. Express each of these statements by a simple English sentence.

(i) $\exists x T(x, \text{Korean}) \land \forall x T(x, \text{Mexican})$
(ii) $\exists x \exists z \forall y (x \neq z \land (T(x, y) \iff T(z, y)))$
3(a) Show that $\forall x (P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ are not logically equivalent.

3(b) Rewrite the following statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

(i) $\neg(\exists x \exists y \neg P(x, y) \land \forall x \forall y Q(x, y))$

(ii) $\neg \exists z \forall y \forall x T(x, y, z)$
4. Use the rules of inference (see next page) to show that the premises $\forall x (P(x) \rightarrow (Q(x) \land S(x)))$ and $\forall x (P(x) \land R(x))$ imply the conclusion $\forall x(R(x) \land S(x))$. Justify each step.