Section 1.5

6. Define the following propositions:

R: It rains.
F: It is foggy.
S: The sailing race will be held.
L: The lifesaving demonstration will go on.
T: The trophy will be awarded.

Now we have:

1. \((\neg R \lor \neg F) \rightarrow (S \land L)\) (premise)
2. \(S \rightarrow T\) (premise)
3. \(\neg T\) (premise)
4. \(\neg S\) (modus tollens on 2, 3)
5. \(\neg S \lor \neg L\) (addition on 4)
6. \(\neg(S \land L)\) (deMorgan’s law on 5)
7. \(\neg(\neg R \lor \neg F)\) (modus tollens on 1, 6)
8. \(\neg R \land \neg \neg F\) (deMorgan’s law on 7)
9. \(R \land \neg F\) (double negation in 8)
10. \(R\) (simplification on 9)

14. (a) Define the statements:

\(C(x)\): \(x\) is a student in this class.
\(R(x)\): \(x\) owns a red convertible.
\(T(x)\): \(x\) has gotten a speeding ticket.

The argument is:

1. \(C(\text{Linda}) \land R(\text{Linda})\) (premise)
2. \(\forall x(R(x) \rightarrow T(x))\) (premise)
3. \(R(\text{Linda}) \rightarrow T(\text{Linda})\) (universal instantiation on 2)
4. \(R(\text{Linda})\) (simplification on 1)
5. \(T(\text{Linda})\) (modus ponens on 3, 4)
6. \(C(\text{Linda})\) (simplification on 1)
7. \(C(\text{Linda}) \land T(\text{Linda})\) (conjunction of 5, 6)
8. \(\exists x(C(x) \land T(x))\) (existential generalization on 7)

(b) Define the statements:
$D(x)$: $x$ has taken a course in discrete mathematics.
$A(x)$: $x$ can take a course in algorithms.

Let the domain for $x$ be the five roommates Melissa, Aaron, Ralph, Veneesha, and Keeshawn.

The argument is:
1. $\forall x D(x)$ (premise)
2. $\forall x (D(x) \rightarrow A(x))$ (premise)
3. $D(c)$ for an arbitrary roommate $c$ (universal instantiation on 1)
4. $D(c) \rightarrow A(c)$ (universal instantiation on 3)
5. $A(c)$ (modus ponens on 3, 4)
6. $\forall x A(x)$ since $c$ was arbitrary (universal generalization on 5)

(c) Define the statements:
$S(x)$: John Sayles produced the movie $x$.
$W(x)$: The movie $x$ is wonderful.
$C(x)$: $x$ is a movie about coal miners.

The domain is all movies. Now the argument is:
1. $\forall x (S(x) \rightarrow W(x))$ (premise)
2. $\exists x (S(x) \land C(x))$ (premise)
3. $S(c) \land C(c)$ (existential instantiation on 2)
4. $S(c) \rightarrow W(c)$ (universal instantiation on 1)
5. $S(c)$ (simplification of 3)
6. $W(c)$ (modus ponens on 4, 5)
7. $C(c)$ (simplification of 3)
8. $W(c) \land C(c)$ (conjunction of 6, 7)
9. $\exists x (W(x) \land C(x))$ (existential generalization on 8)

(d) Define the statements:
$C(x)$: $x$ is in this class.
$F(x)$: $x$ has been to France.
$L(x)$: $x$ has visited the Louvre.

Now the argument is:
1. $\exists x (C(x) \land F(x))$ (premise)
2. $\forall x (F(x) \rightarrow L(x))$ (premise)
3. $C(c) \land F(c)$ (existential instantiation on 1)
4. $F(c) \rightarrow L(c)$ (universal instantiation on 2)
5. $F(c)$ (simplification of 3)
6. $L(c)$ (modus ponens on 4, 5)
7. $C(c)$ (simplification of 3)
8. $C(c) \land L(c)$  
9. $\exists x(C(x) \land L(x))$  

16. (a) This is correct; it is basically modus tollens.
(b) Incorrect. Issac’s car could be fun to drive while also not being a convertible.
(c) Incorrect. Even if Quincy likes all action movies, he may also like some other movies too. (I don’t know whether *Eight Men Out* is an action movie, but this is irrelevant for the argument as it is given.)
(d) This is correct; it is basically modus ponens.

20. (a) This argument is invalid. The two premises have the form $\forall x(p(x) \rightarrow q(x))$ and $q(a)$. It does not follow that $p(a)$ is true.
(b) This argument is valid. It is a case of universal instantiation together with modus ponens.

24. The problem is with lines 3 and 5, which do not follow from 2 as claimed. Also, line 7 is not formed correctly, and makes no sense.

28. One possible argument is:
1. $\forall x(P(x) \lor Q(x))$ (premise)
2. $\forall x(\neg P(x) \land Q(x)) \rightarrow R(x))$ (premise)
3. $P(c) \lor Q(c)$ for arbitrary $c$ (universal instantiation on 1)
4. $(\neg P(c) \lor Q(c)) \rightarrow R(c)$ (universal instantiation on 2)
5. $(\neg P(c) \land Q(c)) \lor R(c)$ (equivalent to 4)
6. $(\neg P(c) \lor \neg Q(c)) \lor R(c)$ (deMorgan’s law on 5)
7. $P(c) \lor \neg Q(c) \lor R(c)$ (double negative in 6)
8. $(P(c) \lor R(c)) \lor \neg Q(c)$ (equivalent to 7)
9. $P(c) \lor (P(c) \lor R(c))$ (resolution on 3, 8)
10. $P(c) \lor R(c)$ (equivalent on 9)
11. $\neg R(c) \rightarrow P(c)$ (equivalent to 10)
12. $\forall x(\neg R(x) \rightarrow P(x))$ since $c$ was arbitrary (universal gen. on 11)

Section 1.6

6. Let $a$ and $b$ be odd numbers. Then there exist integers $k, l$ such that $a = 2k + 1$ and $b = 2l + 1$. We now have

$$ab = (2k + 1)(2l + 1) = 4kl + 2l + 2k + 1 = 2(2kl + l + k) + 1,$$
which is the required form of an odd number. That is, we have shown that 
\( ab = 2m + 1 \) for an integer \( m \) (namely, \( m = 2kl + l + k \)). Hence \( ab \) is odd.

8. We are told that \( n \) is a perfect square, and want to prove that \( n + 2 \) is not a perfect square. We will assume that \( n + 2 \) is a perfect square and derive a contradiction; this will prove that our assumption is false (and the theorem we want is true).

So we are assuming that \( n \) and \( n + 2 \) are both perfect squares. This means that there are positive integers \( a, b \) such that \( a^2 = n \) and \( b^2 = n + 2 \). Our intuition here is that these numbers are too close together to both be squares. Let’s make this precise.

Since \( n < n + 2 \), taking square roots we get \( a < b \). Now consider the number \( (a + 1)^2 = a^2 + 2a + 1 \). Since \( a \) is a positive integer, \( a \geq 1 \), and so

\[
(a + 1)^2 \geq a^2 + 2 + 1 = (n + 2) + 1 = b^2 + 1.
\]

So we now have

\[
a^2 < b^2 < (a + 1)^2.
\]

Taking square roots we get \( a < b < a + 1 \). But this is impossible if \( a \) and \( b \) are both integers. This contradiction shows that \( n \) and \( n + 2 \) cannot both be squares.

15. We are proving that if \( x + y \geq 2 \) then \( x \geq 1 \) or \( y \geq 1 \).

Suppose the conclusion is false. Then, \( x < 1 \) and \( y < 1 \). Adding these, we get \( x + y < 1 + 1 = 2 \), and therefore the hypothesis “\( x + y \geq 2 \)” is false. This proves the theorem.