1. (8 points) Find the following partial derivatives.

(a) $f_x$ and $f_y$ where $f(x, y) = x^y$

\[
\begin{align*}
    f_x &= yx^{y-1} \\
    f_y &= x^y \ln(x)
\end{align*}
\]

(b) $u_{yx}$ where $u = xy^2e^y$

\[
\begin{align*}
    u_y &= xy^2e^y + 2xye^y \\
    u_{yx} &= y^2e^y + 2ye^y
\end{align*}
\]

(c) $f_{yxyxyy}$ where $f(x, y) = x \tan(y^3)$

\[
\begin{align*}
    f_{yxyxyy} &= f_{xxx} \gamma \gamma \gamma \gamma \gamma \gamma \gamma \\ 
    f_x &= \tan(3y) \\
    f_{xx} &= 0 \\
    \text{so } f_{yxyxyy} &= 0.
\end{align*}
\]

(d) $\frac{\partial z}{\partial x}$ where $z = f(xy)$

\[
\frac{\partial z}{\partial x} = f'(xy)y \by \text{the one-variable chain rule.}
\]
2a. (6 points) Sketch the domain of the function \( f(x, y) = \ln(y) + 4\sqrt{x-y} - |xy| \). Be sure to indicate which boundary points are in the domain by using dotted/solid lines, and open/closed dots at special points.

\[ x-y \geq 0 \quad \text{and} \quad y > 0 \]

2b. (2 points) Find the range of \( g(x, y) = |x \sin(y)| \).

\[
\text{range} = \left[ 0, \infty \right)
\]

3. (9 points) A contour map for a function \( f(x, y) \) is shown below.

(a) Estimate \( f(-3, 3) \) and \( f(3, -2) \).

\[
\begin{align*}
  f(-3, 3) &\approx 56 \\
  f(3, -2) &\approx 35
\end{align*}
\]

(b) Estimate \( f_x(0, -3) \) and \( f_y(0, -3) \).

\[
\begin{align*}
  f_x(0, -3) &= 0 \\
  f_y(0, -3) &\approx 30
\end{align*}
\]

(c) Is \( f_{xx}(0, -3) \) positive, negative, or zero?

\[
\text{negative}
\]

\[
( y = -3 \text{ slice is concave down })
\]
4a. (6 points) Consider the function \( f(x, y) = \frac{x^4}{x^2 + y^2} \). Write down a continuous function which agrees with \( f(x, y) \) away from \((0, 0)\). Then find \( \lim_{(x,y) \to (0,0)} \frac{x^4}{x^2 + y^2} \) and explain why the limit exists.

Away from \((0,0)\), \( f(x,y) = \frac{x^2}{1 + y^2} \), which is cont.

Hence \( \lim_{(x,y) \to (0,0)} f(x,y) = \lim_{(x,y) \to (0,0)} \frac{x^2}{1 + y^2} \).

Since \( 1 + y^2 \) is continuous, we can plug in \((0,0)\) to get the limit.

Hence the limit is \( \frac{0}{1 + 0^2} = 0 \).

4b. (6 points) Show that the limit \( \lim_{(x,y) \to (0,0)} \frac{2xy^2}{x^2 + y^4} \) does not exist. [Hint: try approaching along a parabola.]

Approaching along the x-axis we have

\[
\lim_{(x,0) \to (0,0)} \frac{2x(0)^2}{x^2 + (0)^4} = \lim_{x \to 0} \frac{0}{x^2} = 0.
\]

Approaching along the parabola \( x = y^2 \) we have

\[
\lim_{(y^2,y) \to (0,0)} \frac{2y^2y^2}{(y^2)^2 + y^4} = \lim_{(y^2,y) \to (0,0)} \frac{1}{1} = 1.
\]

Two different limits \( \Rightarrow \lim_{(x,y) \to (0,0)} \frac{2xy^2}{x^2 + y^4} \) does not exist.
5. (8 points) Let \( V = \pi r^2 h \). Find \( dV \). Use this to estimate the amount of tin in a closed tin can with diameter 8 cm and height 10 cm if the tin is 0.03 cm thick.

\[
dV = \frac{\partial V}{\partial r} \, dr + \frac{\partial V}{\partial h} \, dh
\]

\[
dV = 2\pi r h \, dr + \pi r^2 \, dh
\]

Thickness = 0.03 cm \( \Rightarrow \)

\[
\begin{align*}
  dr &= 0.03 \text{ cm} \\
  dh &= 0.06 \text{ cm}
\end{align*}
\]

Amount of tin \( \approx dV = 2\pi (4)(10)(0.03) + \pi (4)^2 (0.06) \)

\[
= 3.36\pi \text{ cm}^3
\]
6a. (6 points) Explain why the function \( f(x, y) = e^x \sin(xy) \) is differentiable at \((0, 2)\). Then find either the linearization \( L(x, y) \) at this point or the equation of the tangent plane (your choice).

First, 
\[
\begin{align*}
    f_x &= e^y \cos(xy) + e^x \sin(xy) \\
    f_y &= e^x x \cos(xy)
\end{align*}
\]

These functions are both continuous everywhere, so \( f(x, y) \) is differentiable.

\[
L(x, y) = f_x(0, 2)(x - 0) + f_y(0, 2)(y - 2) + f(0, 2)
\]

\[
L(x, y) = 2x
\]

(tangent plane : \( z = 2x \))

6b. (3 points) Estimate \( f(0.01, 1.97) \). [A calculator should not be necessary.]

\[
f(0.01, 1.97) \approx L(0.01, 1.97) = 0.02
\]