2.5

28. Find the derivative of the function \( f(x) = \frac{\cos(\pi x)}{\sin(\pi x) + \cos(\pi x)} \).

\[
f(x) = \frac{\cos(\pi x)}{\sin(\pi x) + \cos(\pi x)} \quad \Rightarrow \quad f'(x) = \frac{(\sin(\pi x) + \cos(\pi x))(-\pi \sin(\pi x)) - \cos(\pi x)(\pi \cos(\pi x) - \pi \sin(\pi x))}{(\sin(\pi x) + \cos(\pi x))^2}
\]

\[
= \frac{-\pi \{(\sin^2(\pi x) + \cos^2(\pi x))\}}{(\sin(\pi x) + \cos(\pi x))^2} = \frac{-\pi}{(\sin(\pi x) + \cos(\pi x))^2}
\]

OR

\[
= \frac{-\pi}{1+2\sin(\pi x)\cos(\pi x)}
\]

2.8

14. At noon, Ship A is 150 km away west of Ship B. Ship A is sailing East at 35 km/h and Ship B is sailing North at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?

If \( z \) is the distance between the ships, we need to find \( \frac{dz}{dt} \) when \( t = 4h \).

\[ z^2 = (150-x)^2 + y^2 \] . So, at 4 pm, \( x = 4(35) = 140 \) and \( y = 4(25) \), making \( z = \sqrt{(150-140)^2 + 100^2} = \sqrt{10000} = 10\sqrt{101} \).

Moreover, \( z^2 = (150-x)^2 + y^2 \) \( \Rightarrow \) \( 2z \frac{dz}{dt} = -2(150-x) \frac{dx}{dt} + 2y \frac{dy}{dt} \).

So \( \frac{dz}{dt} = \frac{1}{z} \left[ (x-150) \frac{dx}{dt} + y \frac{dy}{dt} \right] = \frac{-10(35) + 100(25)}{10\sqrt{101}} = \frac{215}{\sqrt{101}} \).

Thus, \( \frac{dz}{dt} \approx 21.4 \text{ km/h} \).
3.7

16. A rectangular storage container with an open top is to have a volume of 10 \( m^3 \). The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

First, \( V = lwh \), so \( 10 = (2w)(w)h = 2w^2h \implies h = \frac{5}{w^2} \).

The cost is \( C(w) = 10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh \). Plugging in our height… \( C(w) = 20w^2 + 36w(5/w^2) = 20w^2 + 180/w \). We now just have to minimize this.

\[
C'(w) = 40w - 180/w^2 = 0 \implies w = \frac{\sqrt{9}}{\sqrt{2}} \text{ is the critical value (one should check this is a minimum), meaning } C\left(\frac{\sqrt{9}}{\sqrt{2}}\right) = 20\left(\frac{\sqrt{9}}{\sqrt{2}}\right)^2 + 180/\left(\frac{\sqrt{9}}{\sqrt{2}}\right) \approx 163.54 \text{ is the cheapest cost.}
\]

34. A poster is to have an area of 180 \( in^2 \) with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area.

First, \( xy = 180 \), so \( y = 180/x \). The printed area is \( A(x) = (x-2)(y-3) \) \( = (x-2)(180/x-3) = 186 - 3x - 360/x \). We simply have to maximize this…

\[
A'(x) = -3 + 360/x^2 = 0 \implies x^2 = 120 \implies x = 2\sqrt{30} \text{ (one should check this is a maximum).}
\]

And, \( y = 180/(2\sqrt{30}) = 90/\sqrt{30} \). The dimensions are \( 2\sqrt{30} \text{ in. and } 90/\sqrt{30} \text{ in.} \).

3.9

34. Find \( f \) from \( f''(x) = 8x^3 + 5 \) given that \( f(1) = 0 \) and \( f'(1) = 8 \).

\[
f''(x) = 8x^3 + 5 \implies f'(x) = 8\left(\frac{x^4}{4}\right) + 5x + C. \text{ So, } f'(x) = 2x^4 + 5x + C. \text{ Given that } f'(1) = 8, \text{ we can solve for } C \ldots f'(1) = 2(1)^4 + 5(1) + C = 8 \implies C = 1. \text{ Thus, } f'(x) = 2x^4 + 5x + 1.
\]
Now for \( f \). \( f'(x) = 2x^4 + 5x + 1 \) \( \Rightarrow \) \( f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x + D \). Given that \( f(1) = 0 \), we can solve for \( D \)...

\[ f(1) = \frac{2}{5}(1)^5 + \frac{5}{2}(1)^2 + (1) + D = 0 \] \( \Rightarrow \) \[ D = -1 - \frac{5}{2} - \frac{2}{5} = -\frac{39}{10}. \]

Thus, \( f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - \frac{39}{10}. \)