Mathematics 3113-005 Name (please print) ______________________

Exam II Form B

March 25, 2011

Instructions: Read the question carefully and make sure that you answer the question given. Give concise answers, but clearly indicate your reasoning. Most of the problems have rather short answers.

I. (8) (a) Write a general solution of $y^{(4)} + 4y'' + 4y = 0$.

(b) The function $\cos(x) + 3x^2$ satisfies the DE $y^{(4)} + 4y'' + 4y = \cos(x) + 12x^2 + 24$. Find a general solution to this DE.

II. (6) A certain mass-spring system has $m = 1$, $c = 2$, and $k = 3$, so can be modeled by the equation $x''+2x'+3x = 0$, where $x = x(t)$.

(a) Find a general solution.

(b) What value of c would make the system critically damped?

(c) Is the system in (a) (with $c = 2$) overdamped, critically damped, or underdamped?

III. (3) Define what it means to say that a collection of functions $\{y_1, y_2, \ldots, y_n\}$ is *linearly independent.*

IV. (5) Write the function $-4\cos(\sqrt{3}t) - 3\sin(\sqrt{3}t)$ in phase-angle form.

V. (10) Recall that in the method of variation of parameters, one uses a trial solution of the form $y_p = u_1y_1 + u_2y_2$, and if we choose u_1 and u_2 to satisfy the two conditions $y_1u_1' + y_2u_2' = 0$ and $y_1'u_1' + y_2'u_2' = f(x)$, then y_p will be a particular solution. In this problem, we will use the method to find a particular solution to the DE $y'' - y = -2e^{-x}$. Some remarks:
(1) You do not need to give the theoretical derivation of the two equations involving u_1' and u_2'. We will simply apply them, with the correct choices of y_1 and y_2, to find a particular solution for this example.
(2) Please be careful to calculate accurately. Miscalculations are likely to make the problem become much more complicated. The correct integrals you need to calculate for this problem are easy ones.
(3) Yes, a solution of this DE can be found rather easily using the method of undetermined coefficients, but that is not relevant to this problem, which concerns the method of variation of parameters.

(a) Write the appropriate form of the trial solution $y_p = u_1y_1 + u_2y_2$ for $y'' - y = -2e^{-x}$ (that is, what functions would y_1 and y_2 be?)

(b) Write the appropriate version of the system of two equations $y_1u_1' + y_2u_2' = 0$ and $y_1'u_1' + y_2'u_2' = f(x)$ for $y'' - y = -2e^{-x}$.

(c) Solve the system of two equations and use the results to find a particular solution of $y'' - y = -2e^{-x}$.

VI. (8) (a) Define a *linear combination* of functions.

(b) State the Principle of Superposition for a DE of order n. Be sure to tell the requirements on the DE (that is, the hypotheses) needed for the Principle of Superposition to apply.

(c) Verify the Principle of Superposition for the first-order linear homogeneous DE $y' + P(x)y = 0$.

(Problems VII and VIII are on the other side of this page.)

VII. (7) Using the trial solution $y_p = Ax\cos(x) + Bx\sin(x)$, carry out the rest of the method of undetermined coefficients to find a particular solution of $y'' + y = \cos(x)$.

VIII. (7) For the following two DE's, a general solution of the associated homogeneous equation is shown (you do not need to show how to find it or check that it is a solution). Using this information, write a trial solution for the method of undetermined coefficients, but do *not* continue on to try to determine the exact coefficients.

(a) $y'' + 4y = \cos(2x)$, $y_c = c_1\cos(2x) + c_2\sin(2x)$.

(b) $y^{(4)} + 4y'' = x^2e^x\cos(2x)$, $y_c = c_1 + c_2x + c_3\cos(2x) + c_4\sin(2x)$.