Mathematics 3113-005 Name (please print) ____________________

Quiz 7 Form B

April 15, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (3) For the system $X' = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} X$, a general solution is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = c_1 e^{2t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_3 e^{-t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

(do not derive this or check this). Write a system of linear equations whose solution (c_1, c_2, c_3) gives x_1, x_2, and x_3 satisfying $x_1(1) = 5$, $x_2(1) = 0$, $x_3(1) = -2$. *Do not solve this system,* just write it down.

II. (3) The system of linear equations

$$\begin{aligned} c_1 + c_2 \qquad &= 10 \\ c_1 \qquad + c_3 &= 12 \\ c_1 - c_2 - c_3 &= -1 \end{aligned}$$

arises in solving one of the homework problems (5.1#26). Use the method of Gauss-Jordan elimination to solve this system. That is, rewrite the system as an "augmented" matrix, then do elementary row operations to obtain the values of c_1, c_2, and c_3 that satisfy the system. The first step, writing the augmented matrix, has already been carried out below, just continue the process from there.

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 10 \\ 1 & 0 & 1 & 12 \\ 1 & -1 & -1 & -1 \end{array}\right] \longrightarrow$$

III. (3) Define an *eigenvalue* of a matrix A, and define an *eigenvector* associated to that eigenvalue. You may use the version of the definitions given in class, or the version given in the book, or any equivalent statement.

IV. (7) (a) Show how to calculate that the eigenvalues of the matrix $P = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$ are -1 and 6.

(b) An eigenvector associated to the eigenvalue -1 is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (do not calculate this or check it). Use this to write out a solution X_1 of the system $X' = PX$.

(c) For the eigenvalue 6, find an associated eigenvector $\begin{bmatrix} a \\ b \end{bmatrix}$.