

Instructions: Give concise answers, but clearly indicate your reasoning.

**I.** Given that

(4) 
$$\lambda^6 + 14\lambda^5 + 83\lambda^4 + 268\lambda^3 + 499\lambda^2 + 510\lambda + 225 = (\lambda + 3)^2(\lambda^2 + 4\lambda + 5)^2,$$

write a general solution of the DE

$$y^{(6)} + 14y^{(5)} + 83y^{(4)} + 268y^{(3)} + 499y'' + 510y' + 225y = 0 .$$

**II.** The function  $\cos(x)$  satisfies the DE  $y'' + y' + y = -\sin(x)$ . Find a general solution.

(5)

**III.** Show that the set of functions  $\{1, 2\sin^2(x), \cos^2(x)\}$  is linearly dependent.

(3)

**IV.** Define what it means to say that a collection of functions  $\{y_1, y_2, \dots, y_n\}$  is *linearly independent*.

(3)