Mathematics 3113-005 Name (please print) ______________________

Quiz 4 Form A

March 4, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (3) Define what it means to say that a collection of functions $\{y_1, y_2, \ldots, y_n\}$ is *linearly independent.*

It means that $c_1y_1 + c_2y_2 + \cdots + c_ny_n = 0$ for constants c_i only when all the c_i are 0.

II. (3) Show that the set of functions $\{1, \sin^2(x), 2\cos^2(x)\}$ is linearly dependent.

Since $\sin^2(x) + \cos^2(x) = 1$, we have $1 \cdot \sin^2(x) + (1/2) \cdot 2\cos^2(x) + (-1) \cdot 1 = 0$. Since there is a nonzero linear combination of these functions that equals the zero function, the set is linearly dependent.

III. (4) Given that

$$\lambda^6 + 2\lambda^4 + 20\lambda^3 + \lambda^2 + 20\lambda + 100 = (\lambda + 2)^2(\lambda^2 - 2\lambda + 5)^2 \ ,$$

write a general solution of the DE

$$y^{(6)} + 2y^{(4)} + 20y^{(3)} + y'' + 20y' + 100y = 0 \ .$$

The roots of the characteristic polynomial are -2, -2, $1\pm2i$, and $1\pm2i$. The first two give the solutions e^{-2x} and xe^{-2x}, and the second two pairs give the solutions $e^x\cos(2x)$, $e^x\sin(2x)$, $xe^x\cos(2x)$, and $xe^x\sin(2x)$. These six solutions are linearly independent, so a general solution is

$$c_1e^{-2x} + c_2xe^{-2x} + c_3e^x\cos(2x) + c_4e^x\sin(2x) + c_5xe^x\cos(2x) + c_6xe^x\sin(2x) \ .$$

IV. (5) The function $\sin(x)$ satisfies the DE $y'' + y' + y = \cos(x)$. Find a general solution.

The associated homogeneous DE is $y'' + y' + y = 0$. Its characteristic polynomial is $\lambda^2 + \lambda + 1$, which has roots $-1/2 \pm (\sqrt{3}/2)i$. Therefore a general solution of the associated homogeneous DE is

$$c_1e^{-x/2}\cos(\sqrt{3}x/2) + c_2e^{-x/2}\sin(\sqrt{3}x/2) \ ,$$

and a general solution of the original nonhomogeneous DE is

$$c_1e^{-x/2}\cos(\sqrt{3}x/2) + c_2e^{-x/2}\sin(\sqrt{3}x/2) + \sin(x) \ .$$