Mathematics 3113-005 Name (please print) ____________________

Quiz 3 Form B

February 25, 2011

Instructions: Give concise answers, but clearly indicate your reasoning.

I. (6) Two linearly independent solutions of the DE $y'' - 3y' + 2y = 0$ are e^x and e^{2x} (do not check these).

(a) Write a general solution of $y'' - 3y' + 2y = 0$.

$y = Ae^x + Be^{2x}$.

(b) Find the solution that satisfies $y(1) = 1$, $y'(1) = 0$.

$y' = Ae^x + 2Be^{2x}$. We want

$$1 = y(1) = Ae + Be^2$$
$$0 = y'(1) = Ae + 2Be^2$$

Subtracting the two equations gives $1 = -Be^2$, so $B = -e^{-2}$. From the first equation $1 = Ae - e^{-2}e^2$, so $A = 2/e$. The desired solution is $2e^x/e - e^{-2}e^{2x} = 2e^{x-1} - e^{2x-2}$.

II. (3) This problem concerns the DE $y'' + 2y' + 2 = -2x$.

(a) Write the associated homogeneous equation of $y'' + 2y' + 2 = -2x$.

$y'' + 2y' = 0$.

(b) A solution of $y''+2y'+2 = -2x$ is $1-x$ (do not check this). Given that $e^{-x}\cos(x)$ and $e^{-x}\sin(x)$ are linearly independent solutions of the associated homogeneous equation, write a general solution of $y''+2y'+2 = -2x$.

$Ae^{-x}\cos(x) + Be^{-x}\sin(x) + 1 - x$.

III. (4) For the DE $9y'' + 9y' + y = 0$, the characteristic equation is $9r^2 + 9r + 1 = (3r+1)^2$. Since it has repeated roots $-1/3$ and $-1/3$, two solutions of the DE are $e^{-x/3}$ and $xe^{-x/3}$ (do not check that they are solutions). Compute the Wronskian of $e^{-x/3}$ and $xe^{-x/3}$.

$(e^{-x/3})' = -e^{-x/3}/3$ and $(xe^{-x/3})' = e^{-x/3} - xe^{-x/3}/3$, so

$$W(e^{-x/3}, xe^{-x/3}) = \det \begin{pmatrix} e^{-x/3} & xe^{-x/3} \\ -e^{-x/3}/3 & e^{-x/3} - xe^{-x/3}/3 \end{pmatrix} = e^{-2x/3} - xe^{-2x/3}/3 - (-xe^{-2x/3}/3) = e^{-2x/3}\ .$$

IV. (2) This problem concerns the DE $y'' + y + x = 0$. The function $\sin(x) - x$ is a solution, but $2(\sin(x) - x)$ is not. Why does this not violate the Principle of Superposition?

The DE is not homogeneous (put into the standard form, it becomes $y'' + y = -x$).