Instructions: Give concise answers, but clearly indicate your reasoning.

I. Two linearly independent solutions of the DE $y'' + 3y' + 2y = 0$ are e^{-x} and e^{-2x} (do not check these).
(6)

(a) Write a general solution of $y'' + 3y' + 2y = 0$.

(b) Find the solution that satisfies $y(1) = 1$, $y'(1) = 0$.

II. This problem concerns the DE $y'' + y - x = 0$. The function $\sin(x) + x$ is a solution, but $2(\sin(x) + x)$ is
(2) not. Why does this not violate the Principle of Superposition?

III. This problem concerns the DE $y'' - 2y' + 2 = 2x$.
(3)

(a) Write the associated homogeneous equation of $y'' - 2y' + 2 = 2x$.

(b) A solution of $y'' - 2y' + 2 = 2x$ is $x + 1$ (do not check this). Given that $e^x \cos(x)$ and $e^x \sin(x)$ are linearly independent solutions of the associated homogeneous equation, write a general solution of $y'' - 2y' + 2 = 2x$.

IV. For the DE $4y'' + 4y' + y = 0$, the characteristic equation is $4r^2 + 4r + 1 = (2r + 1)^2$. Since it has repeated
(4) roots $-1/2$ and $-1/2$, two solutions of the DE are $e^{-x/2}$ and $xe^{-x/2}$ (do not check that they are solutions). Compute the Wronskian of $e^{-x/2}$ and $xe^{-x/2}$.